Agricultural Economics Research

APRIL 1966 Vol. XVIII, No. 2

in this issue

Page

UNITED STATES DEPARTMENT OF AGRICULTURE • Economic Research Service

CONTRIBUTORS

LUTHER G. TWEETEN is Associate Professor of Agricultural Economics at Oklahoma State University. FRED H. TYNER is an Agricultural Economist with the Farm Production Economics Division of ERS, stationed at Oklahoma State University.

HERMAN BLUESTONE is Acting Head, Poultry Section, Commodity Analysis Branch, Economic and Statistical Analysis Division, ERS. ANTHONY S. ROJKO was formerly Head, Price Research and Methods Section, Commodity Analysis Branch, Economic and Statistical Analysis Division, ERS. He is now with the World Analysis Group, Foreign Regional Analysis Division, ERS. In his new position, Mr. Rojko coordinates the Division's work on the long-range outlook for supply and demand for agricultural products in foreign countries.

THOMAS A. MILLER is an Agricultural Economist with the Farm Production Economics Division, ERS, stationed at Iowa State University.

JOHN E. LEE, JR., is an Agricultural Economist in the Production Adjustments Branch, Farm Production Economics Division, ERS.

REX F. DALY is Chairman of the Outlook and Situation Board, ERS.

ROBERT M. WALSH is Deputy Director, Economic and Statistical Analysis Division, ERS.

B. RALPH STAUBER is Chief of the Agricultural Price Statistics Branch, SRS.

H. ALBERT GREEN, Leader of the Economic Structures Group, Economic Development Division, ERS, is currently working on techniques for analyzing regional economic structures and growth.

ROBERT K. WINTERS is Director of the International Forestry Staff of the Forest Service.

Agricultural Economics Research

A Journal of Economic and Statistical Research in the United States Department of Agriculture and Cooperating Agencies

APRIL 1966 Vol. XVIII, No. 2

The Utility Concept of Net Social Cost--A Criterion for Public Policy

By Luther G. Tweeten and Fred H. Tyner

AN OPTIMUM FARM POLICY is expected to meet many standards. Farmers want high income, consumers want low food cost, and taxpayers want low Treasury cost. Given the current economic structure of agriculture, these goals are mutually exclusive and lead to conflicts in formulating public policies. Higher farm income means higher food or Treasury cost; lower food cost means reduced farm income or increased Treasury cost; and lower Treasury cost means decreased farm income or expanded consumer food cost. Clearly a criterion for public policy that transcends these individual, narrow goals would be welcome.

In this paper we develop the utility concept of net social cost as a general criterion for policy. This criterion has many limitations to be discussed subsequently. It is a supplement, not a replacement for other criteria used to judge the desirability of specific programs.

Social cost has been used frequently by agricultural economists as a criterion for judging the merits of a particular farm commodity program, market structure, or resource allocation (4, 15, 18, 21, and 22).[1] The social cost concept has been viewed as the net value of goods and services foregone by producing either too much or too little of a particular commodity. The concept cannot be defended on the simplified ground that more of a good (or service) is preferred to less, because too much of any particular commodity can be undesirable. It may be defended on the ground that a larger "basket" of goods and services is preferred to a smaller one, but then aggregation problems emerge. The need arises to compare the worth of having more of one versus another commodity. In this and other ways, social cost involves interpersonal and intercommodity measures of value. These measures are cardinal, and, we believe, entail the implicit assumption that a low social cost has more utility than a high social cost, even where the concept is expressed in dollars. We proceed on this premise, that the social cost concept is implicitly and fundamentally tied to cardinal utility measures.

A review of numerous writings has not uncovered an adequate exposition of the relationship between utility of individuals and the social cost concept derived from industry demand and supply (1-3, 5-14, 16, 17). This paper is at least a partial effort to alleviate the confusion which surrounds the use of industry demand and supply to measure social cost, and the relationship of social cost to utility of consumers.

The concept of social cost developed here may be criticized as a "regression" to Benthamite notions of cardinal utility measurement. Following Pareto and the principle of "Occam's razor" (economy in use of assumptions to reach a given conclusion), the modern trend has been to deemphasize cardinal measures in favor of ordinal measures of utility (cf. 17). Perhaps the trend has gone too far.

The analysis is methodological, intended to illustrate the assumptions and possible uses and limitations of cardinal utility as a pedagogic device and as an added criterion for public policy. The next section, a mathematical development of the utility concept, is followed by an empirical application to the U.S. wheat market.

Social Cost Criterion

The concept of social benefit is developed from individual consumer utility functions. Consider for a single consumer a domain of two commodities, q_1 and q_2, selling at constant prices P_1 and P_2, respectively. Given the utility function (1) associated with consumption of the two goods, the consumer's welfare function U^* is specified as (2).

[1] Underscored numbers in parentheses refer to items in the Literature Cited, p. 41.

$$(1)\quad U = U(q_1, q_2)$$

$$(2)\quad U^* = U(q_1, q_2) + \lambda(Y_0 - P_1 q_1 - P_2 q_2)$$

The welfare function specifies total utility subject to the income restraint $Y_0 = P_1 q_1 + P_2 q_2$ and with λ a Lagrangian multiplier.[2] To maximize utility, the derivatives with respect to q_1, q_2, and λ are computed and set equal to zero in (3) to (5):

$$(3)\qquad \frac{\partial U^*}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda P_1 = 0$$

$$(4)\qquad \frac{\partial U^*}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda P_2 = 0$$

$$(5)\qquad \frac{\partial U^*}{\partial \lambda} = Y_0 - P_1 q_1 - P_2 q_2 = 0$$

If (1) were an explicit utility function, the marginal utilities in (3) and (4) would be specified and (3), (4), and (5) could be solved for λ and the utility-maximizing levels of q_1 and q_2 (5).

The total derivative of (1) with respect to q_1 and q_2 gives the following:

$$(6)\qquad \frac{dU}{dq_1} = \frac{\partial U}{\partial q_1} + \frac{\partial U}{\partial q_2}\frac{dq_2}{dq_1}$$

$$(7)\qquad \frac{dU}{dq_2} = \frac{\partial U}{\partial q_2} + \frac{\partial U}{\partial q_1}\frac{dq_1}{dq_2}$$

The first right-hand term is the direct marginal utility, and the second is indirect marginal utility derived from changing consumption of the other good. Assuming that the marginal utility of q_1 is independent of q_2 and the marginal utility of q_2 is independent of q_1, the second right-hand terms in (6) and (7) drop out, and (3) and (4), without being solved simultaneously, may be specified as:

$$(8)\qquad \frac{dU}{dq_1} = \lambda P_1, \text{ and}$$

$$(9)\qquad \frac{dU}{dq_2} = \lambda P_2.$$

Assume that the marginal utility of money λ is constant and arbitrarily assigned a value $\lambda = 1$. Constant utility of money units is likely to be approached only for small changes in consumption of q_1 or q_2, or if the commodity in question (q_1 for our purposes) represents a small part of the consumer's purchases. Given these assumptions, the marginal utility of consuming q_1 is measured by its price, i.e.:

$$(10)\quad \frac{dU}{dq_1} = P_1 \text{ or } dU = P_1 dq_1$$

The demand function (11) is formed by solving (3) to (5) for q_1.

$$(11)\quad q_1 = D(P_1) \text{ or } P_1 = D^{-1}(q_1)$$

The demand quantity is a function of price P_1 (with P_2 and Y_0 fixed) as specified by the demand function (11). Substituting (11) for the demand price P_1 in (10), the integral from 0 to n is the total utility (12) from consuming n units of q_1.

$$(12)\qquad U = \int_0^n D^{-1}(q_1)dq_1$$

Since (11) becomes a marginal utility curve under the stated assumptions, the integral (12) measures total utility and is the area underneath the demand curve. The integral can be formed only if the demand function is continuously defined and touches the price axis. The assumption that price (and marginal utility) is finite even for a quantity approaching zero seems reasonable, especially if q_1 is not a necessity and if substitutes exist. Since the total utility from consuming q_1 is measured by the area beneath the demand curve from $q_1 = 0$ to $q_1 = n$, as n becomes larger the entire area beneath the consumer demand curve is included.

Equation (12) can be aggregated over all consumers to form the total demand and utility functions for q_1 if, for each consumer, the marginal utility of a given quantity of q_1 is independent of the quantities consumed by others (absence of external economies or diseconomies in consumption). If the independence condition is satisfied, and λ is homogeneous for all consumers, then the area beneath the market demand curve is a measure of utility gained or total social benefit from consuming q_1.

Consumption of q_1 not only gives direct utility measured by (12), but also involves a cost of

[2] Income can be regarded as the flow of goods and services from assets (resources) over a specified period with the asset position remaining the same at the end as the beginning of the period. Hence consumer utility is maximized subject to the asset or resource distribution.

utility foregone by consuming q_1 rather than other commodities represented by q_2. To determine the utility foregone, it is necessary to specify a production function (13) for outputs q_1 and q_2 from the variable input x.[3] The private cost for a firm is the resource price P_x multiplied by the quantity, or $P_x x$. The firm profit π in (14) is maximized by equating derivatives of the expression (15) to zero in (16) to (19). The Lagrangian multiplier is designated as μ. Equations (13) to (19) are:

$$(13) \qquad F(q_1, q_2, x) = 0$$

$$(14) \qquad \pi = P_1 q_1 + P_2 q_2 - P_x x$$

$$(15) \qquad \pi^* = P_1 q_1 + P_2 q_2 - P_x x + \mu F(q_1, q_2, x)$$

$$(16) \qquad \frac{\partial \pi^*}{\partial q_1} = P_1 + \mu \frac{\partial F}{\partial q_1} = 0$$

$$(17) \qquad \frac{\partial \pi^*}{\partial q_2} = P_2 + \mu \frac{\partial F}{\partial q_2} = 0$$

$$(18) \qquad \frac{\partial \pi^*}{\partial x} = -P_x + \mu \frac{\partial F}{\partial x} = 0$$

$$(19) \qquad \frac{\partial \pi^*}{\partial \mu} = F(q_1, q_2, x) = 0$$

Rearranging terms and dividing (16) by (17), (18) by (16), and (18) by (17), the respective results are (20), (21), and (22):

$$(20) \qquad -\frac{dq_2}{dq_1} = \frac{P_1}{P_2}, \text{ or } P_1 = -\frac{dq_2}{dq_1} P_2$$

$$(21) \qquad \frac{dq_1}{dx} = \frac{P_x}{P_1}, \text{ or } P_1 = \frac{dx}{dq_1} P_x$$

$$(22) \qquad \frac{dq_2}{dx} = \frac{P_x}{P_2}, \text{ or } P_2 = \frac{dx}{dq_2} P_x$$

Equations (20) and (21) are two expressions of q_1 cost. Expression (21) indicates that P_1 is equal to the direct private cost of a small increment in q_1. $P_x dx$ is the increment in total cost associated with dq_1, hence $P_x dx/dq$ is the marginal cost. It is apparent that the supply price P_1 in the firm supply function may then be regarded as a measure of marginal cost.

P_1 viewed from the production or firm side in (20) is a measure of the opportunity cost or value of production (and consumption) foregone by producing additional q_1. The amount of production foregone dq_2 divided by an increment dq_1 and multiplied by the price P_2 is the value of production sacrificed. The two expressions of cost $-\frac{dq_2}{dq_1} P_2$ and $\frac{dx}{dq_1} P_x$ are equal.

Again assuming the marginal utility of money is unity ($\lambda = 1$), from (9) we derive $P_2 = dU/dq_2$. This expression for the marginal utility of q_2 is inserted into (20) giving:

$$(23) \qquad P_1 = -\frac{dq_2}{dq_1}\frac{dU}{dq_2} = -\frac{dU}{dq_1}, \text{ or } dU = -P_1 dq_1$$

The relationship between supply price and quantity is specified by the supply function (24), found by solving (16) to (19) for q_1:

$$(24) \qquad q_1 = S(P_1), \text{ or } P_1 = S^{-1}(q_1)$$

After substituting (24) for P_1 in (23), the total utility foregone by production and consumption of q_1 is specified in (25) by integrating (23) over the range 0 to n:

$$(25) \qquad U = -\int_0^n S^{-1}(q_1) dq_1$$

The integral is the area beneath the individual firm supply curve. It is a valid measure of total utility foregone only if the marginal utility of money is constant, the marginal cost curve is continuously defined, and there is no divergence between social and private cost.

Under competitive conditions and excluding external economies or diseconomies of scale in production,[4] the individual firm marginal

[3] Resources designated x are variable in the length of run considered. Other fixed resources will influence the productivity coefficients, hence the resulting marginal conditions and utility are subject to the initial distribution of assets in both production and consumption.

[4] Some costs are external to the firm but internal to the industry. In the long run, many such costs are reflected in private accounts of the firm. More important is the disassociation between private and social costs (or returns) that do not become reflected in private accounts of the firm even as the length of run increases.

cost (supply) functions can be aggregated to form the industry supply curve. The area beneath the industry supply function is a measure of the total utility foregone or opportunity cost of producing and consuming q_1 rather than q_2 under the conditions stated above.

The total utility U_T or net social gain from consumption and production of q_1 is the sum of the direct utility (12) and the utility foregone (25), i.e., (26):

$$(26) \quad U_T = \int_0^N D^{-1}(Q_1)dQ_1 - \int_0^N S^{-1}(Q_1)dQ_1$$

where Q_1 and N designate industry demand and supply relationships. To maximize U_T, we take the derivative with respect to Q_1 in (26) and set it equal to zero. The solution is the quantity at which the supply price and demand price (marginal utilities) are equal. That is, q_1 increases until the satisfactions achieved from consuming it just equal the satisfactions foregone by not consuming other commodities represented by q_2. Since this occurs only at the intersection of supply and demand, it follows that prices and quantities under perfect competition maximize utility, subject to the initial resource distribution.

Additional assumptions are that knowledge is complete, products and resources are mobile, and second-order conditions of convexity, etc., are met. Given these conditions, the equilibrium specified from (26) with price P_1 and P_2 represents a Pareto optimum.

Dividing (8) by (9), the result (27) is equivalent to (20):

$$(27) \quad -\frac{dq_2}{dq_1} = \frac{P_1}{P_2}$$

The marginal rate of substitution of q_1 for q_2 in production for all firms and in consumption for all consumers equals the same price ratio, thus they are equal to each other--a necessary condition for a Pareto optimum.

A two-commodity world of q_1 and all other commodities denoted by q_2 was considered above to simplify and shorten the analysis, but the results also apply when a larger group of commodities is included.

ADDITIONAL ASSUMPTIONS

Figure 1 illustrates the concepts developed mathematically. D is industry market demand; S is industry supply. Gross social benefit is the area beneath D; gross social cost (variable cost) is the area beneath S. Gross social benefit less social cost is the net social gain, i.e., the sum of areas A, r, s, t, and u. Net social gain is divided into two portions: consumer surplus, i.e., the area s, r, and part of A above p_e; and producer surplus (profit), i.e., area t, u, and part of A below p_e. The net social cost c utility foregone by underproducing at q_a rather than the competitive equilibrium q_e is measured by the triangle A. The net social cost of overproduction at q_b is triangle B.

The concept of net social cost may become more clear with an intuitive argument. At any given wheat quantity, the vertical distance from the quantity axis to the demand curve is one measure of the social benefits of that quantity, and the distance to the supply curve is one measure of the social cost. It follows that the difference between these vertical segments, the distance between the demand and supply curves, is one measure of the net social gain from producing and consuming the particular quantity

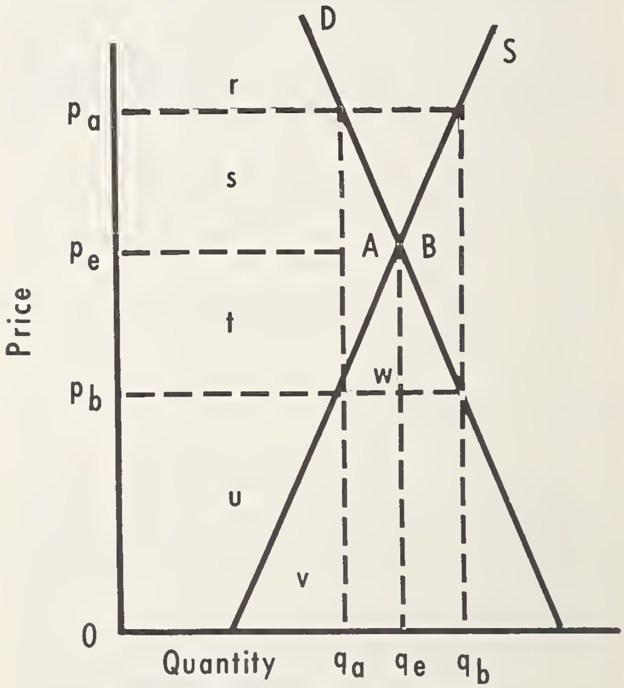

Figure 1.--Hypothetical illustration of the social cost concept.

of (say) wheat. If we sum the net social gains for each bushel of wheat, the area between the supply and demand curves is traced. Positive additions to social gain are made by moving to the right until supply and demand intersect. This is the equilibrium output in a "free" market, unrestricted by production controls and price supports. It follows that the equilibrium price and quantity established in free markets maximize the net social gain, given the initial resource distribution. Competitive equilibrium is a "local" utility maximization or Pareto optimum on a contract curve.[5] Movement along the contract curve to a "global" utility maximum requires a redistribution of income or resources, based on value judgments and generally resolved through voting or other social mechanisms.

Sometimes, net social gain might be judged to be divided inequitably between producer surplus and consumer surplus. In that event it may be argued that, since the equilibrium price and quantity (at the intersection of supply and demand) represent the largest "pie" of utility available, consumers can compensate producers adequately and still have more "pie" than at any other output. Further, the free market equilibrium output may represent the most acceptable output and resource return for producers and consumers alike simply because it arises from an impersonal pricing mechanism which society may prize for rewarding factors according to their contribution.

The applicability of this argument for agriculture has been questioned, particularly in regard to the following conditions: (a) the initial distribution of assets, (b) transferability of resources to alternative uses with a minimum of cost, time, and friction, and (c) the degree to which decision-makers are informed about returns to resources in alternative uses. If some farmers are poorly endowed with resources at the outset, free markets may not allow them to accumulate a socially acceptable income in a reasonable period. Labor in an industry sometimes receives depressed earnings because of an inelastic demand coupled with demand contraction or supply expansion. If institutional and psychological restraints delay the transfer of labor to more lucrative employment, returns may be depressed over extended periods. Price and output at the intersection of supply and demand in such an industry need not maximize national welfare. Some believe that agriculture is such an industry, in which welfare or satisfaction is not necessarily maximized by the output and prices resulting from the intersection of demand and supply without Government regulation of the market.

Another assumption of the mathematical development of net social cost is absence of external economies or diseconomies of scale. This assumption may be quite innocuous in the wheat example presented later but can be very important in other applications.

Perhaps the most serious limitation arises in the application of the net social cost concept to farm programs requiring sizable Government transfer payments to farmers. These may be regarded as transfers of consumer surplus to producers in compensation for an unacceptably low producer income. In fact, however, it is not consumers of the specific commodity but all taxpayers who provide the transfer payment.

The public may judge that the sum of the areas A, r, s, t, and u to the left of demand and supply in figure 1 (the total potential net gain) is divided inequitably between farmers and consumers, and a program to redistribute the net social gain is initiated. Taxes or output restrictions redistributing income to farmers change output from q_e to q_a or q_b. The expense for personnel and equipment to administer the program is likely to be a social cost. The tax and subsidy may represent a social cost to the extent that the marginal utility of money is higher among taxpayers than among farmers. If marginal utilities of money are equal for taxpayers and farmers, then the social cost of a program that redistributes income by transfer payments through the U.S. Treasury will be small, providing output does not deviate appreciably from q_e. The fact that the income redistribution is approved by society may imply a higher marginal utility of money to farmers than to taxpayers--hence a conceivable utility gain from a tax-subsidy program. The subsequent discussion abstracts from measuring the social gain or loss from Government expenditures because of obvious measurement problems.

[5] For definition of the contract curve see Melvin Reder (16, p. 23).

National programs to redistribute income can shift the industry demand curve and, more likely, the supply curve. Farmers invest income supports in capital improvements and improved technology (21). The result may increase supply and bid resources away from uses more favored by society, thereby reducing utility. But purchase of capital representing improved technology may increase productivity and aggregate utility. The net effect on utility is unknown.

The exact validity of the social cost concept discussed above also requires equilibrium in perfectly competitive markets for all commodities for a fixed level of q_1. Given equilibrium in nonperfect markets for commodities other than q_1 (as for agriculture vis-a-vis other economic sectors), the net social cost in figure 1 for q_1 will remain essentially valid, representing maximum net utility attainable under the given structure. It is clear that the net social gain concept must be used with caution. Relevance depends partially on the commodity being examined.

Application of Social Cost Concept to Wheat Programs

The purpose of this paper is to examine the social cost concept, and not to analyze the wheat market structure. Thus little background is given on wheat markets and implications of alternative wheat programs (19 and 20). The following estimates do, however, give some insight into the social cost of several wheat programs (designated I to XI in this paper) that are current contenders in the policy milieu. Social cost is the area A or B in figure 1. It is here measured in dollars of goods foregone because of a nonoptimum output, but it could be assigned a utility index by setting a value (say, one dollar equals one unit of utility) as the marginal utility of money λ.

Programs in table 1 are characterized by low Government cost. Net farm income is increased above the free market level by exercise of monopoly control of wheat production and market allocation to maximize net farm income.

FREE MARKET

Without supply controls or Government price and income support, the equilibrium wheat price is $1.20, production 1,480 million bushels. Net income to farmers is $787 million, and social cost is low.

Table 1.--Wheat industry pricing and market allocation under competitive (unrestricted) and supply control (monopoly) market structures[1]

Item	Free market (unrestricted, production program I)	Supply control or monopoly: One-price program (II)[2]	Two-price program (III)[3]	Three-price program (IV)[4]
Food, Seed, Industry:				
Price....dol./bu..	1.20	1.25	2.00	2.00
Quantity....mil. bu..	565	563	545	545
Returns....mil. dol..	681	704	1,090	1,090
Feed:				
Price....dol./bu..	1.20	1.25	1.22	1.19
Quantity....mil. bu..	135	94	119	144
Returns....mil. dol..	163	118	145	172
Exports:				
Price....dol./bu..	1.20	1.25	1.22	1.23
Quantity....mil. bu..	780	643	724	699
Returns....mil. dol..	939	804	885	860
Gross wheat receipts...mil. dol..	1,783	1,620	2,120	2,122
Total production cost..mil. dol..	996	801	891	891
Net farm returns.......mil. dol..	[5] 787	819	1,229	1,231
Total quantity.........mil. bu..	1,480	1,296	1,388	1,388
Planted acres.......... mil..	[5] 66.3	53.4	59.4	59.4
Yield per planted acre.......bu..	22.3	24.2	23.4	23.4
Social cost..........mil. dol..	0	26	14	15

[1] Prices, output, costs, and returns are at the farm level. Totals may not be exact because of rounding.
[2] The equilibrium quantity is determined by equating the marginal revenue computed from the aggregate demand function, with marginal cost (supply). The individual market allocation is found by computing the demand quantity in each market at the price $1.25.
[3] The equilibrium quantity is determined by summing the two marginal revenue curves of (a) the domestic food, seed, and industry market, and (b) the feed and foreign export market, and equating the combined function to marginal cost (supply). The equilibrium marginal revenue is related back to the component demand, with the price and quantity in each major market specified by the equilibrium marginal revenue.
[4] The same procedure as in footnote 3, but with 3 markets.
[5] Some of the wheat production with free markets may simply replace feed grain with little change in net returns on acres where this substitution occurs. If 60 million acres is the effective acreage, excluding substitutions, the net return under free markets is $706 million.

MULTIPLE PRICE PROGRAMS

Net farm income is not much greater under the one-price monopoly program than under the free market because of the highly elastic total demand for wheat at low prices when wheat becomes competitive with feed grains at home and abroad. We arbitrarily specify under all wheat programs that the domestic food wheat price can be no higher than $2 per bushel. Social cost of the one-price plan is high ($26 million) compared to other programs in table 1 because considerable consumer surplus potential is foregone in feed wheat and export markets at $1.25 per bushel.

Net farm income is considerably enhanced under the two-price plan, where the domestic food, seed, and industry market is separated

from the feed and export market. Social cost is $14 million, or approximately half the social cost of the one-price plan. Where demand is considerably elastic, as for wheat in the feed and export markets, a small increase in price substantially raises the social cost.

There appear to be few advantages to the three-price plan over the two-price plan, according to table 1. Income is a little higher. Social cost is up only slightly, but administration and other problems of separating the three markets would probably rule out the program.

DIRECT PAYMENTS

Voluntary programs involving Government supports are compared in table 2 with the two-price monopoly program. To facilitate comparisons, all programs are adjusted to give the same net farm income as program III. Programs are defined more fully in table 2 footnotes which refer to Government cost assumptions illustrated in figures 2 and 3. Under the direct payment program V, pricing and output are the same as under the free market. Direct income supplements are used to raise net farm income from wheat to the prescribed $1,229 million.

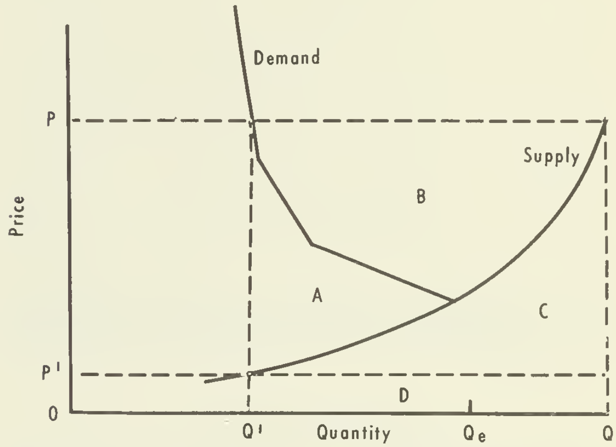

Figure 2.--Hypothetical examples of Government costs with voluntary acreage diversion programs operated at different levels of efficiency.

Table 2.--Implications of selected programs in achieving a prescribed net farm income from wheat of $1,229 million

Item	Supply control--two-price plan (III)[1]	Voluntary programs							
		Direct payment-lump sum (V)[2]	Acreage diversion		Market subsidies				
					Allotments		No allotments		
			Efficient (VI)[3]	Less efficient (VII)[4]	Efficient (VIII)[5]	Less efficient (IX)[6]	Efficient (X)[7]	Less efficient (XI)[8]
Price....................dol./bu..	1.22-2.00	1.20	1.73	1.28	1.50	1.50	1.49	1.49
Quantity.................mil. bu..	1,388	1,480	703	1,184	1,480	1,480	1,570	1,570
Market returns..........mil. dol..	2,120	1,783	1,212	1,519	2,225	2,225	2,340	2,340
Government payments......mil. dol..	--	442	386	414	[9] 50	[9] 232	[9] 166	[9] 271
Gross returns.........mil. dol..	2,120	2,225	1,598	1,933	2,225	2,225	2,340	2,340
Total nonland cost.......mil. dol..	891	996	369	704	996	996	1,111	1,111
Net farm returns.......mil. dol..	1,229	1,229	1,229	1,229	1,229	1,229	1,229	1,229
Planted acres...............mil...	59.4	66.3	24.6	46.9	66.3	66.3	74.3	74.3
Yield per planted acre.........bu..	23.4	22.3	28.6	25.2	22.3	22.3	21.1	21.1
Treasury cost [10]........mil. dol..	Small	442	386	414	50	232	116	271
Income increment above free market per unit Treasury cost........................dol..	Large	1.00	1.15	1.07	8.84	1.91	2.66	1.63
Social cost.............mil. dol..	14	Small	386	75	Small	Small	8	8

[1] See the two-price plan, table 1. Throughout the table, data may not be exact because of rounding.
[2] The difference between the free market equilibrium in table 2 and the prescribed income is made up by a direct payment to farmers. This payment must be independent of future production or equilibrium prices and quantities will change as well as other implications above.
[3] Through market and production contract discrimination, the Government cost is assumed to be area A, figure 2.
[4] More realistic than the "efficient" program, Government cost is area ABCD, figure 2.
[5] Allotments are at the free market level, 66.3 million acres. Government cost is A, figure 3. The Government cost or subsidy is not paid directly to farmers, but is included indirectly in market receipts.
[6] Government costs are AC in figure 3 with $Q = Q_e$.
[7] Government cost is A, figure 3.
[8] Government cost is ABC, figure 3.
[9] The Government costs are included in farm receipts, thus need not be added as in other cases to gross farm returns.
[10] Does not include administration and storage cost.

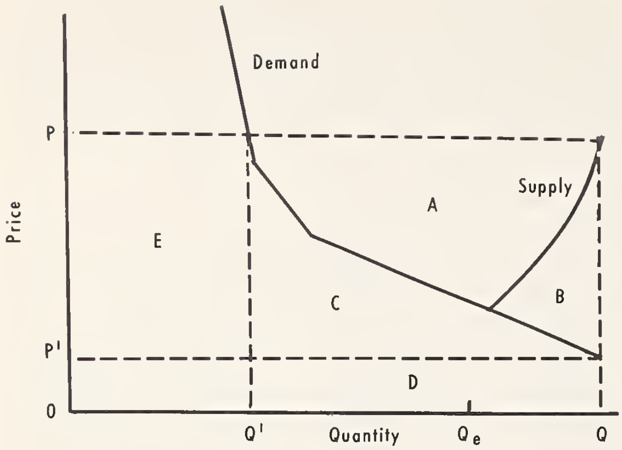

Figure 3.--Hypothetical examples of Government costs with market subsidy programs operated at different levels of efficiency.

Assuming that payments can be independent of expected price and other inducements that would change output from the free market level, the social cost of program V is nominal. Payments could be tied to past allotment history, the farm, or individuals. Whether these payment schemes would in fact leave output at the free market equilibrium is questionable, however.

ACREAGE DIVERSION

Acreage diversion programs have been used to control feed grain production in recent years, and have been used to a small extent for wheat. Under acreage diversion programs, the Government pays farmers to remove land from wheat production--to convert part of their wheat allotment to soil conserving uses. Program VI shows that if the Government uses sealed bids and other means to administer the program efficiently[6] and make each Government dollar go far in raising farm income with an acreage diversion program, the social cost is very high. The reason is that production must be curtailed severely before the quantity is cut from the elastic portion of the wheat market (below about $1.40 per bushel) to the inelastic domestic food portion of the demand. The acreage diversion program VI alone does not appear to be acceptable for maintaining wheat prices at a high level.

If the Government administers the program less efficiently, with greater transfer payments per unit of production removed, the income supplement from Government payments helps raise farm income with less acreage cutback under VII than under VI. Because wheat is priced competitively with feed grains, program VII could be combined in a joint feed grain-wheat program. Social cost is $75 million--somewhat greater than under the multiple price programs in table 1.

MARKET SUBSIDY

Market subsidy programs require a subsidy equal to the difference between the market support price and the demand price. Given supply and demand, the extent to which markets are discriminated determines the subsidy required. Market subsidy programs can be administered by issuing Government subsidies to exporters as necessary to move desired quantities, or the Government can first purchase quantities in excess of market needs at the desired support price, then export the excess at whatever terms are feasible.

Prices supported above competitive equilibrium encourage overproduction. Income advantages of market subsidies over free markets might be used as incentives for farmers to minimize social cost by restricting output to the free market level (programs VIII and IX, table 2). With full market discrimination, paying a subsidy equal to the difference between the support price and the demand price on each bushel, Government cost could be as low as $50 million. In practice, this is not possible. A subsidy on all bushels (except domestic food) equal to the difference between the support and demand price on the last bushel would result in a more realistic level of Government cost, $232 million in program IX.

Without allotments, farmers would produce an estimated 1,570 million bushels on 74.3

[6] Here the terms "efficient" and "inefficient" do not refer to waste or mismanagement in administering programs, but rather to the extent of efforts to pay individual producers the minimum required to curtail production or pay individual processors the minimum subsidy between market support price and demand price for utilizing wheat. The decision to administer the program without sealed bids and market discrimination may be a conscious and planned effort to avoid friction and ease administration problems.

million acres at the $1.49 support price. Again with perfect market discrimination, Government cost would be $166 million in program X, social cost only $8 million. Social cost remains unchanged but Government costs are raised to $271 million with the same subsidy paid on all wheat (except domestic food) in program XI. Government cost would be $487 million if the difference of $0.31 between the support price ($1.49) and the demand price ($1.18) were paid on all production, including that utilized in domestic food markets. But assuming that the large Government transfer payments from taxpayers to farmers were between individuals with the same marginal utility of money, social cost would be the same in programs X and XI.

SUMMARY AND CONCLUSIONS

Programs to restrict output such as supply control and acreage diversion involve the greatest social cost. Of programs designed to provide wheat farmers with $1,229 million net income, direct payments, market subsidies, supply control (two-price monopoly), and acreage diversion programs rank from lowest to highest in social cost. In general, however, social cost is not large in relation to net farm income. Social cost, as a proportion of net farm income, is only 6.1 percent for acreage diversion program VII, 1.1 percent for supply control program III, and 0.7 percent for market subsidy programs X and XI. It is apparent that supply control programs such as III could involve greater social cost than some Government programs.

Social costs of redistributing income to producers tend to be low if demand is either perfectly elastic or perfectly inelastic. Social cost of a two-price plan even with a sizable redistribution of income is not large with a combination of a highly inelastic demand (domestic food) and a highly elastic export and feed demand. Allocation to the domestic food market is not changed markedly from the free market because quantity is not responsive to the higher price--hence social cost is small. Marginal revenue in the feed-export market approaches the nearly horizontal demand curve--thus the intersection of the marginal revenue curve (monopoly) and the demand curve (competitive) with the supply curve occurs at nearly the same output, especially if the supply curve is steep.

The social cost is sensitive to the export demand specification. For example, when the export demand curve is made to fall twice as fast as that used in tables 1 and 2, social cost is increased respectively to $82, $33, and $35 million for programs II, III, and IV.[7]

In conclusion, the concept of social cost in this study appears promising for some uses but has limitations. The application to wheat markets is not exact because assumptions are violated and estimating techniques are imperfect. But we believe the concept does give useful insight into utility foregone by over- or under-extending output. The criterion supplements but does not replace estimates of farm income, Government cost, consumer cost, freedom in production and marketing, and income increments per Government dollar in appraising policy alternatives.

Literature Cited

(1) Boulding, K. E. The concept of economic surplus. Amer. Econ. Rev., Vol. 35, No. 5, pp. 851-869, December 1945.

(2) Fisher, Irving. Mathematical investigations in the theory of value and prices. Appendix I, Yale Univ. Press, New Haven, 126 pp., 1925. (Originally pub. in Vol. 9, Transactions of the Conn. Academy, 1892).

(3) Friedman, Milton, and L. J. Savage. The utility analysis of choices involving risk. In: Readings in Price Theory, K. E. Boulding and G. J. Stigler (eds.), pp. 57-96, Richard D. Irwin, Chicago, 1952.

(4) Gustafson, R. L. Carryover levels for grain. U.S. Dept. Agr., Tech. Bul. 1178, 92 pp., 1958.

(5) Henderson, J. M., and R. E. Quandt. Microeconomic theory: A mathematical approach. McGraw-Hill, New York, 291 pp., 1958.

(6) Hicks, J. R. The rehabilitation of consumers' surplus. Rev. Econ. Studies, Vol. 8, No. 2, pp. 108-121, February 1941.

(7) Hicks, J. R. The four consumers' surpluses. Rev. Econ. Studies, Vol. 11, No. 1, pp. 31-41, Winter 1943.

[7] Unfortunately, lack of space precludes discussion of the many qualifications and limitations on the demand and supply estimates. The reader is again referred to Tweeten (19).

(8) Hicks, J. R. The generalised theory of consumer's surplus. Rev. Econ. Studies, Vol. 13, No. 2, pp. 68-74, 1945-46.

(9) Hotelling, Harold. Demand functions with limited budgets. Econometrica, Vol. 3, pp. 66-78, January 1935.

(10) Hotelling, Harold. The general welfare in relation to problems of taxation and of railway and utility rates. Econometrica, Vol. 6, No. 3, pp. 242-269, July 1938.

(11) Hotelling, Harold. The relation of prices to marginal costs in an optimum system. Econometrica, Vol. 7, No. 2, pp. 151-155, April 1939.

(12) Knight, Frank. Realism and relevance in the theory of demand. Jour. Pol. Econ., Vol. 52, No. 4, pp. 289-318, 1944.

(13) Marshall, Alfred. Appendix H and mathematical appendix. Principles of Economics, Macmillan, New York, 871 pp., 1952 ed. (1890).

(14) Mishan, E. J. Realism and relevance in consumer's surplus. Rev. Econ. Studies, Vol. 15, No. 37, pp. 27-33, 1947-48.

(15) Parish, R. M. The costs of protecting the dairying industry. Econ. Record, Vol. 38, No. 82, pp. 167-182, June 1962.

(16) Reder, Melvin W. Studies in the theory of welfare economics. Columbia Univ. Press, New York, 208 pp., 1947.

(17) Samuelson, Paul A. Foundations of economic analysis. Harvard Univ. Press, Cambridge, Mass., pp. 208-210, 1947.

(18) Takayama, T., and G. G. Judge. Spatial equilibrium and quadratic programming. Jour. Farm Econ., Vol. 46, No. 1, pp. 67-93, February 1964.

(19) Tweeten, Luther G. Commodity programs for wheat. Okla. Agr. Expt. Sta. Tech. Bul. T-118, 1965.

(20) Tweeten, Luther G., and Fred K. Hines. Markets and programs for wheat. Okla. Current Farm Econ., Vol. 38, pp. 18-27, April 1965.

(21) Tweeten, Luther G., and James S. Plaxico. Long-run outlook for agricultural adjustments based on national growth. Jour. Farm Econ., Vol. 46, No. 1, pp. 39-55, February 1964.

(22) Wallace, T. D. Measures of social costs of agricultural programs. Jour. Farm Econ., Vol. 44, No. 2, pp. 580-594, May 1962.

Forecasting Farm Turkey Prices In and Out of the Main Marketing Period

By Herman Bluestone and Anthony S. Rojko

TURKEY PRODUCERS need to be able to forecast turkey prices with a high degree of accuracy if they are to make sound and profitable production and marketing decisions. For instance, good price forecasts can help a producer decide (1) whether to market his birds at light weights as fryer-roasters or to carry them to maturity for later marketing, (2) whether to sell his birds live or to have them custom processed and stored for later sale in ready-to-cook form, or (3) whether to contract to grow turkeys for a price specified in advance.

The object of this study is to provide equations for more accurately forecasting the price of turkeys to producers in and out of the heavy marketing period. These equations can be used for forecasting prices only when the level of poultry supplies is known in advance. Supplies can be estimated 3 to 6 months in advance from cold storage holdings, the number of poults and chicks hatched, and the number of eggs in incubators. If price forecasts further into the future are needed, as they would be for production planning, then some method of forecasting supplies for the corresponding period would be needed.

Turkey is traditionally a holiday bird. Consumption reaches seasonal peaks at Thanksgiving and Christmas. Because of this and because conditions are most favorable for producing turkeys with spring-hatched poults, a large proportion of turkey production is geared for marketing in the last few months of the year. Turkey marketings begin the year at a very low level and increase steadily to a high in the fourth quarter. They begin exceeding consumption around midyear. Cold storage holdings of turkey build up from midyear to a peak in November to provide maximum supplies for the holiday period.

In the last 5 years, a rapidly increasing proportion of turkey meat has been used in the production of convenience foods such as turkey roasts and rolls, reaching about 15 percent in 1964. To date this development has had no material effect on seasonal patterns of marketings but its continuance could reduce the seasonal pattern.

Despite the highly seasonal production and consumption pattern, the few statistical studies designed to measure turkey demand and the factors influencing turkey prices have employed annual time series data.[1] This approach implies that the impact of factors influencing changes in the demand for turkey from one year to the next, such as population growth, competition from other foods, and per capita disposable income, is distributed throughout each year in roughly the same way. Analysts were aware that the demand might be somewhat different in different periods of the year. However, data were not available for measuring these differences.

Because of the large proportion of the crop marketed in September-December, turkey prices during this period usually average rather close to the annual average (fig. 1). Thus, price forecasts from analyses using annual data would be expected to be better indicators of prices in the main marketing season than in the January-August period.

However, in recent years new data have become available which make it possible to develop analyses for periods of less than a year (tables 1 and 2). This report presents results from a study using these new data to evaluate and measure separately turkey demand in the periods of heavy and light marketings. Analyses for shorter periods might have been more useful for indicating the best time for marketing

---

[1]Karl A. Fox, "The Analysis of Demand for Farm Products," U.S. Dept. Agr., Tech. Bul. 1081, 90 pp., 1953. G. E. Brandow, "Interrelations Among Demands for Farm Products and Implications for Control of Market Supply," Pa. Agr. Expt. Sta., Bul. 680, 124 pp., 1961. Dennis Lee Bawden, "Interregional Models of the United States Turkey Industry," Ph.D. diss., Univ. Calif., 1964. Olan D. Folker, "The 1965 Turkey Outlook," speech presented at Natl. Turkey Fed. Conv., Des Moines, Iowa, 13 pp., 1965.

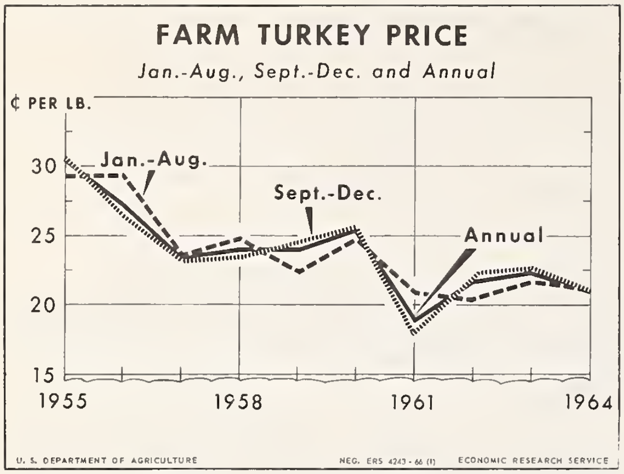

Figure 1

Table 1.--Data for January-August analyses[1]

Year	Per capita supplies available for domestic consumption in Jan.-Aug.[2] Chicken C_{J-A}	Turkey T_{J-A}	Per capita red meat consumption in Jan.-Sept. M_{J-S}	Per capita disposable income in Jan.-Sept. Y_{J-S}	Consumer price index in Jan.-Aug. (1957-59 = 100)	Weighted farm turkey price per pound in Jan.-Aug. deflated by CPI P_{J-A}
	Pounds	Pounds	Pounds	Dollars	Index	Cents
1955....	14.1	1.9	119.1	1,644	93.2	31.4
1956....	16.0	2.0	123.8	1,729	94.2	31.1
1957....	17.0	2.8	118.0	1,803	97.5	24.3
1958....	19.0	2.6	111.9	1,817	100.5	24.7
1959....	20.4	2.8	117.0	1,890	101.2	22.1
1960....	19.2	2.8	120.4	1,934	102.8	24.0
1961....	21.2	3.7	118.9	1,971	104.0	20.1
1962....	20.7	3.5	121.2	2,054	105.1	19.2
1963....	21.3	3.2	124.9	2,113	106.4	20.3
1964....	21.8	3.4	129.1	2,234	107.9	19.5
1965[3]...	22.8	3.4	124.7	2,371	109.5	18.7

[1] Data in analyses were estimates as of mid-1965. [2] Estimated production in January-August plus beginning stocks minus exports. [3] Estimates based on preliminary data available in January 1966.

Table 2.--Data for September-December analyses[1]

Year	Per capita supply of turkey available for domestic consumption in Sept.-Dec. Total A_{S-D}	From commercial sources T_{S-D}	Change in per capita consumption of chicken and turkey in Jan.-Aug. from a year earlier ΔQ_{J-A}	Per capita red meat consumption in Oct.-Dec. M_{O-D}	Time (1955 = 1) X	Consumer price index in Sept.-Dec. (1957-59 = 100)	Per capita disposable income in Oct.-Dec. deflated by CPI Y_{O-D}	Weighted farm turkey price per pound in Sept.-Dec. deflated by CPI P_{S-D}	Per capita turkey consumption from commercial sources in Sept.-Dec. U_{S-D}
	Pounds	Pounds	Pounds	Pounds		Index	Dollars	Cents	Pounds
1955......	4.1	4.1	-1.1	43.7	01	93.6	1,816	32.6	3.5
1956......	4.6	4.4	1.8	42.9	02	95.9	1,848	27.5	3.5
1957......	4.8	4.8	1.9	40.7	03	98.9	1,824	23.6	3.7
1958......	4.9	4.8	1.5	39.7	04	100.8	1,836	23.2	3.8
1959......	4.9	4.8	1.6	42.5	05	102.2	1,869	24.0	3.9
1960......	4.9	4.8	-1.2	40.4	06	103.7	1,873	24.7	3.9
1961......	6.3	5.9	2.4	41.6	07	104.6	1,934	17.1	4.5
1962......	5.6	5.4	-.5	41.9	08	106.0	1,961	20.9	4.3
1963......	5.6	5.4	.5	44.4	09	107.3	2,012	21.1	4.2
1964......	5.8	5.5	.5	44.9	10	108.6	2,108	19.4	4.4
1965[2].....	6.0	5.8	1.1	42.3	11	110.6	2,214	17.7	4.7

[1] Data in analyses were estimates as of mid-1965.
[2] Estimates based on preliminary data available in January 1966.

birds. However, estimates of supply for shorter periods, a necessary variable in the analyses, would be considerably less reliable. The study provides statistical formulas for predicting turkey prices in and out of the main marketing period and for the year as a whole. It seeks to provide answers to such questions as these: Are the factors affecting turkey prices in September-December the same as those in January-August? Do changes in turkey supplies affect turkey prices more when marketings are seasonally heavy or light? Does the lateness of the turkey crop, that is, changes in the proportion of the crop marketed in September-December, affect the annual average turkey price?

Major Findings

The demand for turkey at the farm level is elastic during January-August but inelastic in

September-December.[2] The study showed that during the last decade (1955-64) the elasticity of demand in January-August approached -2.0 while during the main marketing period it averaged around -0.5. For the year as a whole price elasticity of demand averaged around -0.7. Elasticities computed with 1964 values were about 30 percent lower in January-August, and 10 to 15 percent lower in September-December and for the year as a whole, than those computed with 1955-64 average values. These seasonal differences confirm that outside the holiday period turkey has to compete much more directly with chicken and other high-protein foods.

Because of these differences in demand, prices in the main marketing period were considerably more responsive to changes in supplies than prices outside the main marketing period. In September-December, turkey prices (in constant dollars) were found to be measurably influenced by only two factors--per capita turkey supplies and change from a year earlier in per capita poultry consumption in January-August. It was found that, other things being equal, an increase of 5 percent in per capita turkey supplies from commercial sources (total supplies excluding USDA purchases) during this period was followed, on the average, by a 10 percent decrease in the turkey price. It was most interesting that year-to-year changes in per capita poultry supplies (including chicken) prior to the main marketing season were significant factors in affecting the September-December price while the absolute level of per capita chicken supplies during this heavy marketing period was not. This strongly suggests that heavy per capita use of poultry in January-August relative to a year earlier tends to weaken demand for turkey in the holiday season.

In January-August, the per capita supply of chicken was the only variable, besides per capita supplies of turkey, to measurably affect deflated turkey prices. During this period a 10 percent increase in the per capita supplies of turkeys resulted in about a 5 percent drop in the deflated price of farm turkeys. A 10 percent increase in chicken supplies depressed turkey prices by about the same amount.

The statistical relations measuring the effect on turkey price of the several supply factors were evaluated as to their adequacy for estimating the price of turkeys. During the period of fit (1955-64) the statistical equation for September-December gave price estimates for each of the 10 years that were within 1 cent of the actual price.[3] For the January-August period, the price estimate deviated from the observed price by more than 1 cent in only 3 of the 10 years, the largest deviation being 1.7 cents. Comparisons were also made to determine whether the annual average price could be estimated more accurately by combining the results from the two regressions for the separate periods or by using a single regression equation fitted with annual data. When using the combined results, it was possible to obtain price estimates within 0.7 cent or less of the observed price. However, the use of the equation based on annual data gave price estimates in 2 of the 10 years that deviated from the observed value by more than 1 cent.

The Model

The main objective of the study was to develop relationships for forecasting farm turkey prices in the January-August period, in the September-December period, and for the year as a whole. A secondary aim was to measure the responsiveness of consumption to changes in prices.

The basic mathematical models developed to forecast prices were essentially of the form

(1) $P = f(T, C, M, Y)$

where P is the farm turkey price, T the supply of turkeys, C the supply of other poultry, M the supply of red meat, and Y consumer income. Quantities and income were converted to a per capita basis so population would not

[2] Even though the major objective of this study was to estimate turkey prices, demand elasticities were computed from these statistical relations to provide comparisons. These elasticities are not necessarily the same as those that would be derived from a statistical model designed to obtain statistical demand coefficients. However, we feel that they are probably not far from such coefficients.

[3] The true test of an estimating equation is how well it will predict future prices.

have to be treated as a separate variable in the analysis. Price and income data were deflated by the consumer price index.

Graphic analysis indicated that January-August relationships appeared to be linear while those for September-December appeared to be nonlinear. For this reason, a semilogarithmic function was used for the September-December period and for the year as a whole.

The mathematical models were fitted by the least-squares method using data for 1955-64.[4] Data for periods of less than a year were not available prior to 1955.

January-August Regressions

The major variables investigated in the analysis of the period of light turkey marketings are defined below:

P_{J-A} = Weighted U.S. farm turkey price, deflated by CPI (cents per pound).

T_{J-A} = Per capita turkey supply available for domestic consumption (pounds).

C_{J-A} = Per capita chicken supply available for domestic consumption (pounds).

M_{J-S} = Per capita red meat consumption (pounds).

Y_{J-S} = Per capita disposable income deflated by CPI (dollars).

All of the data were for January-August, except per capita red meat consumption and per capita disposable income which were for January-September.

The investigation revealed that per capita chicken supplies and per capita turkey supplies explained 96 percent of the variation in the deflated farm turkey price in January-August 1955-64. When red meat, disposable income, and time were included as additional explanatory factors, the relationship was not materially improved. The relevant price estimating equation obtained is equation (2):

$$(2)\quad P_{J-A} = 50.315 - \underset{-3.3}{4.21}\, T_{J-A} - \underset{-2.6}{0.76}\, C_{J-A}$$

$$R^2 = 0.96 \qquad D.W. = 2.64 \qquad S.E. = 1.014$$

The numbers under the regression coefficients are the "t" statistics.[5] Both coefficients as indicated by their "t" values are significant at the 5 percent confidence level. The standard error of estimate of P_{J-A} is 1.0 cent. The Durbin-Watson statistic (D.W. = 2.64) reveals that probably little autocorrelation exists in the residuals.

The results of the analysis seem reasonable. Turkey supplies in January-August are small both in absolute terms and in relation to other high-protein supplies; therefore, it might be expected that only chicken, the closest substitute for turkey, would affect turkey prices enough to be clearly measurable. Then too, the impact of income on turkey prices probably might be diluted and hard to identify specifically, since turkey makes up such a small part of the total poultry supply in January-August.

Price flexibilities computed from equation (2) indicate that a 10 percent change in per capita turkey supplies available for domestic consumption in January-August from commercial sources, all other things being equal, was followed by only about a 5 percent change in deflated farm turkey prices in the opposite direction. However, if the price flexibility is computed with data using 1964 values rather than the average values of the variables for the period, a 10 percent change in supplies results in a 7 percent change in prices. The reciprocals of these price flexibility coefficients are 2.0 and 1.4, respectively. They imply a direct price elasticity of demand for turkey that is greater than one. It is logical to believe that demand for turkey in January-August is elastic because turkey supplies are small and turkey is a close substitute for many different high-protein foods.

In January-August, a 5 1/2-pound change in chicken supplies has about the same effect on turkey prices as a 1-pound change in turkey supplies. However, when changes in supplies are measured in percentage terms, chicken has

[4] The use of a single equation rather than a system of simultaneous equations appears to be valid for price forecasting since the supplies of turkey, chicken, and red meat are determined prior to the marketing period and are not influenced much by the current farm turkey price.

[5] The "t" value is the ratio of the regression coefficient to its standard error and is used to ascertain whether or not the coefficient differs significantly from zero.

about the same impact on prices as turkey. This is not too surprising when one considers that chicken supplies in January-August in recent years have been 6 to 8 times as large as turkey supplies. In the last decade, a 10 percent change in chicken supplies was, on the average, associated inversely with a 6 percent change in turkey prices. When 1964 values rather than the average values for the period were used, the computed price flexibility increased to -0.8.

September-December Regressions

Price-supply relationships for this period, because of the larger marketings, are perhaps of greater interest than those for the January-August period. The variables studied during this main turkey marketing period included:

A_{S-D} = Per capita turkey supply available for domestic consumption from all sources, September-December (pounds).

T_{S-D} = Per capita turkey supply available for domestic consumption from commercial sources, September-December (pounds).

ΔQ_{J-A} = Change in per capita consumption of chicken and turkey from a year earlier, January-August (pounds).

M_{O-D} = Per capita red meat consumption, October-December (pounds).

X = Time (1955=1).

Y_{O-D} = Per capita disposable income deflated by CPI, October-December (dollars).

P_{S-D} = Weighted farm turkey price deflated by CPI, September-December (cents per pound).

U_{S-D} = Per capita turkey consumption from commercial sources, September-December (pounds).

Using these variables for 1955-64, the following price-estimating equations were statistically developed.

$$(3)\quad \text{Log } P_{S-D} = 1.947 \underset{-12.2}{-\ 0.112}\, A_{S-D} \underset{-\ 1.7}{-\ 0.008}\, \Delta Q_{J-A}$$

$R^2 = 0.96$ D.W. = 1.66
S.E. = 0.0174 (in logarithms)

$$(4)\quad \text{Log } P_{S-D} = 2.042 \underset{-17.6}{-\ 0.135}\, T_{S-D} \underset{-\ 2.8}{-\ 0.009}\, \Delta Q_{J-A}$$

$R^2 = 0.98$ D.W. = 1.86
S.E. = 0.0122 (in logarithms)

$$(5)\quad \text{Log } P_{S-D} = 1.736 \underset{-\ 5.3}{-\ 0.115}\, T_{S-D} \underset{-\ 2.7}{-\ 0.012}\, \Delta Q_{J-A} \underset{-\ 1.3}{-\ 0.0073}\, X + \underset{1.1}{0.0013}\, Y_{S-D}$$

$R^2 = 0.99$ D.W. = 2.50
S.E. = 0.0125 (in logarithms)

In contrast to the January-August analysis, the September-December analysis considers two separate supply variables--supplies available for domestic consumption from all sources, and supplies available for domestic consumption from commercial sources (excluding USDA purchases). The USDA has purchased turkeys during most of the years used in the analysis. These purchases always have been timed to affect prices in the main marketing period.

The use of per capita turkey supplies available for domestic consumption from commercial sources in the analysis appeared to give a better fit than when supplies from all sources were used--for example, equation (4) versus equation (3). These supplies and the year-to-year change in per capita chicken and turkey consumption in January-August, T_{S-D} and ΔQ_{J-A}, alone explained 98 percent of the variation in deflated turkey prices (equation 4) during the period under investigation. In addition, the regression coefficients associated with these two variables remained stable and highly significant even when other explanatory variables were added to the regressions. Analysis

revealed that the absolute level of per capita chicken consumption in September-December had little measurable effect on turkey prices. The Durbin-Watson statistic (D.W.) for equations (3) and (4) indicates no serial correlation exists.

When "demand shifters"--per capita disposable income and time--were introduced, the percentage of the explained variation in the dependent variable increased somewhat, but neither of the coefficients associated with these two "shifter" variables were significant at the 5 percent confidence level (equation 5).

Some regressions were also run with per capita consumption of turkey from commercial sources as the dependent variable to permit direct estimating of elasticity of demand.[6]

$$(7)\quad \text{Log } U_{S\text{-}D} = 0.827 \underset{-3.0}{-0.011}\, \Delta Q_{J\text{-}A} \underset{-9.2}{-0.010}\, P_{S\text{-}D}$$

$R^2 = 0.92$ D.W. $= 2.22$
S.E. $= 0.124$ (in logarithms) E $= 0.57$

$$(8)\quad \text{Log } U_{S\text{-}D} = 0.631 \underset{-2.0}{-0.008}\, \Delta Q_{J\text{-}A} \underset{-5.0}{-0.0080}\, P_{S\text{-}D} \underset{1.2}{+0.00008}\, Y_{O\text{-}D}$$

$R^2 = 0.94$ D.W. $= 2.16$
S.E. $= 0.0119$ (in logarithms) E $= 0.43$

[6]Since retail turkey prices were not available, farm prices (prices paid by processors, rather than prices paid by consumers) were used for this purpose. Dealer demand may be used to represent consumer demand if a relatively fixed relationship between farm and retail prices can be assumed, or if a separate shift variable is used to represent marketing activity. The relation between farm and retail prices does appear to be fairly stable. In nearly all of the January-August period, freshly killed turkeys compete directly with turkeys being taken out of cold storage for sale to consumers. And nearly all of the turkeys slaughtered in September-December are consumed during that period. In any year, processors may misjudge consumer demand in the fall and pay too much or too little to farmers. Also, retailers may misjudge consumer demand. Even so, there probably might be as much tendency to err on the high side as on the low side and thus the average relationship between farm and retail prices for a period of years might not be affected much.

Demand during the main marketing season is indicated to be inelastic as would be expected. When the average values of the economic variables are used, the price elasticity of demand is about -0.5.[7] That is, a change in per capita turkey consumption from commercial sources of about 5 percent is associated with a 10 percent change in deflated farm turkey prices in the opposite direction. The elasticity dropped to -0.4 when 1964 values were used in computing the elasticity coefficient.

Annual Regressions

Time series relationships for the year as a whole were fitted to provide a basis of comparison for the January-August and September-December analyses.

The variables analyzed included (see table 3):

A = Total per capita turkey supplies available for domestic consumption (pounds).

T = Per capita turkey supplies available for domestic consumption from commercial sources (pounds).

ΔC = Change from a year earlier in per capita chicken consumption (pounds).

[7]The formula for obtaining the price elasticity of demand for a semilog function of the form

$$(1)\qquad \text{Log}_{10}\, q = bp$$

where q is consumption and p is price, can be derived as follows:

The general formula for elasticity is

$$(2)\qquad E = \frac{dq}{dp} \cdot \frac{p}{q}$$

Differentiating equation (1) with respect to q, we get

$$(3)\qquad \frac{dq}{dp} = bq \log_e 10$$

$$(4)\qquad \frac{dq}{dp} = 2.3026\ bq$$

and substituting in (2):

$$(5)\qquad E = 2.3026\ bq \cdot \frac{p}{q} = 2.3026\ bp$$

M = Per capita red meat consumption (pounds).

X = Time (1955 = 1).

Y = Per capita disposable income deflated by CPI (dollars).

P = Weighted farm turkey price deflated by CPI (cents per pound).

U = Per capita consumption of turkey from commercial sources (pounds).

Two variables, T (per capita turkey supplies for the year as a whole) and ΔC (change from a year earlier in per capita chicken consumption), were found to influence the deflated annual farm turkey price. The regression follows:

$$(9) \quad \text{Log } P = 1.966 - \underset{-21.9}{0.082\, T} - \underset{-2.3}{0.005\, \Delta C}$$

$R^2 = 0.99$ D.W. = 1.28
S.E. = 0.0103 (in logarithms)

Per capita turkey supplies excluding USDA purchases, as in the September-December analyses, gave a much better fit than total per capita supplies available for domestic consumption. The addition of income, time, and meat as explanatory variables did not improve the relationship as was true for the September-December analysis.

When per capita consumption of turkey from commercial sources was treated as the dependent variable, the deflated farm turkey price, P, and the change from a year earlier in per capita chicken consumption proved to be the only significant independent variables. The Durbin-Watson test reveals no serial correlation in the residuals.

$$(10) \quad \text{Log } T = 1.087 - \underset{-6.2}{0.009\, \Delta C} - \underset{-26.7}{0.013\, P}$$

$R^2 = 0.99$ D.W. = 2.18
S.E. = 0.0060 (in logarithms)

During the period under study, about a 7 percent change in the annual per capita turkey consumption from commercial sources was associated with a 10 percent change in deflated farm turkey prices in the opposite direction. The elasticity computed for 1964 was -0.6. Thus, the demand for turkey in the year as a whole is less elastic than in January-August and almost as inelastic as in September-December, as would be expected.

Table 3.--Data for annual analyses, 1955-65[1]

Year	Per capita supply of turkey available for domestic consumption: Total[2] A	Per capita supply of turkey available for domestic consumption: From commercial sources[3] T	Change in per capita chicken consumption from a year earlier ΔC	Per capita red meat consumption M	Time (1955 = 1) X	Consumer price index (1957-59 =100)	Per capita disposable income deflated by CPI Y	Weighted farm turkey price deflated by CPI P	Per capita turkey consumption from commercial sources[4,5] U
	Pounds	*Pounds*	*Pounds*	*Pounds*		*Index*	*Dollars*	*Cents*	*Pounds*
1955......	5.7	5.7	-1.5	162.8	01	93.3	1,779	32.4	5.0
1956......	6.3	6.1	3.1	166.7	02	94.7	1,839	28.7	5.0
1957......	7.0	7.0	1.1	158.7	03	98.0	1,840	23.9	5.9
1958......	6.9	6.8	2.6	151.6	04	100.7	1,813	23.7	5.8
1959......	7.2	7.1	.8	159.5	05	101.5	1,876	23.5	6.1
1960......	7.1	7.0	-.8	160.8	06	103.1	1,878	24.6	6.0
1961......	8.9	8.6	2.1	160.5	07	104.2	1,904	18.1	7.1
1962......	8.2	8.0	-.2	163.1	08	105.4	1,954	20.5	6.8
1963......	8.0	7.8	.8	169.3	09	106.7	1,992	20.9	6.5
1964......	8.4	8.1	.4	174.6	10	108.1	2,082	19.4	6.9
1965[6].....	8.6	8.4	2.2	168.5	11	109.9	2,232	18.5	7.2

1 Data in analyses were estimates as of mid-1965.
2 Production plus beginning stocks less exports.
3 Total supply available for domestic consumption minus USDA purchases.
4 Civilian disappearance estimated from production, stock changes, exports and military use.
5 Civilian disappearance minus USDA purchases.
6 Estimates based on preliminary data available in January 1966.

Estimating Turkey Prices

Turkey prices in the period of heavy and light marketings have differed considerably in some of the years in the last decade. Figure 1 shows weighted average turkey prices for three periods, January-August, September-December, and the year as a whole. A much closer relationship exists between September-December prices and annual prices than between January-August prices and prices in the other two periods. Because of this, price-estimating equations from studies with annual aggregates in the past have been used for estimating changes in the September-December price. However, such September-December price estimates have an estimating error inherent in the equations themselves as well as an additional error arising from price variation within the year.

Table 4 and figures 2 and 3 compare the price estimates obtained from the price-estimating equation with the observed prices during September-December and January-August for 1955-64. With equation (4) it was possible to estimate prices for September-December that came within 0.9 cent of the observed price in each of the 10 years and within 0.5 cent of it in 7 years. Less accurate price estimates were made for January-August. The estimated price for this period deviated from the observed price by more than 1 cent in 3 years, the largest deviation being 1.7 cents in 1961.

Table 4.--January-August and September-December farm turkey price, reported and estimated, 1955-65

Year	January-August price			September-December price		
	Reported weighted average	Estimated by equation (2)[1]	Deviation	Reported weighted average	Estimated by equation (4)[2]	Deviation
	Cents	*Cents*	*Cents*	*Cents*	*Cents*	*Cents*
1955....	29.3	29.4	0.1	30.5	29.6	-0.9
1956....	29.3	28.0	-1.3	26.4	26.0	-.4
1957....	23.7	24.9	1.2	23.3	23.6	.3
1958....	24.8	25.0	.2	23.4	24.3	.9
1959....	22.4	23.2	.8	24.5	24.5	0
1960....	24.7	24.6	-.1	25.6	25.4	-.2
1961....	20.9	19.2	-1.7	17.9	17.6	-.3
1962....	20.2	20.8	.6	22.2	22.2	0
1963....	21.6	21.9	.3	22.6	21.9	-.7
1964....	21.0	20.9	-.1	21.1	21.4	.3
1965[3]...	22.4	20.5	-1.9	22.1	19.6	-2.5

[1] Equation (2): $P_{J-A} = 50.315 - 4.21\ T_{J-A} - 0.76\ C_{J-A}$
[2] Equation (4): $\text{Log}\ P_{S-D} = 2.042 - 0.135\ T_{S-D} - 0.009\ \Delta Q_{J-A}$
[3] Based on preliminary data available in January 1966.

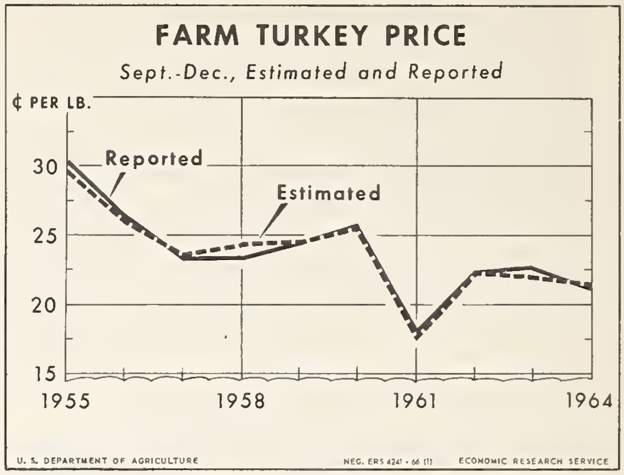

Figure 2

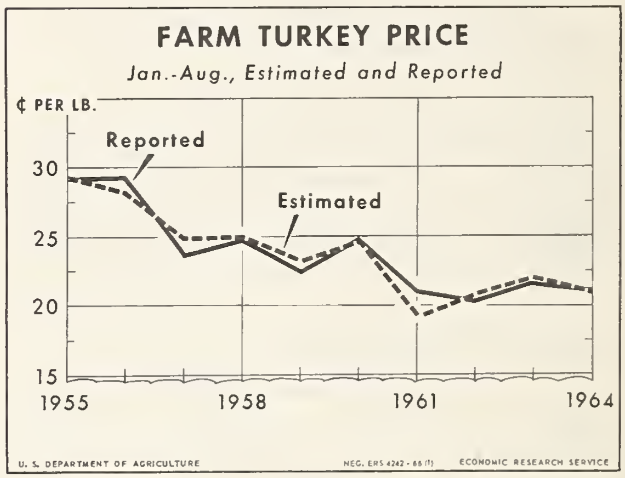

Figure 3

Table 5 shows that estimates of the annual average farm turkey price made by weighting the January-August and September-December price estimates obtained from equations (2) and (4) were better than annual estimates made directly from equation (9) which was based on annual data. Estimated prices based on the two equations deviated from observed prices by 0.7 cent or less in each of the years in the study and by less than 0.5 cent in 5 years. Equation (9) yielded estimates that deviated from observations by more than 1 cent in 2 years.

Tables 4 and 5 also show how prices estimated for 1965 by equations (2), (4), and (9) compare with reported prices. Estimated prices are much too high. The overestimates appear to have resulted from the substantially reduced supplies of red meat, especially pork. In addition, there was a large increase in consumer income in 1965. Normally, most of the year-to-year price variation can be explained without allowing for the effect of these two factors. However, in 1965, both worked in the same direction and their combined effect appears to explain why prices were above those predicted by the equations.

Table 5.--Annual farm turkey price, reported and estimated, 1955-65

Year	Reported weighted average price	Estimated by equation (9)[1]		Estimated by weighting estimates from equations (2) and (4)	
		Price	Deviation	Price	Deviation
	Cents	*Cents*	*Cents*	*Cents*	*Cents*
1955......	30.2	29.6	-0.6	29.5	-0.7
1956......	27.2	25.9	-1.3	26.5	-.7
1957......	23.4	24.2	.8	24.0	.6
1958......	23.9	25.0	1.1	24.5	.6
1959......	23.9	24.3	.4	24.1	.2
1960......	25.4	25.5	.1	25.1	-.3
1961......	18.9	18.3	-.6	18.2	-.7
1962......	21.6	21.4	-.2	21.8	.2
1963......	22.3	22.2	-.1	21.9	-.4
1964......	21.0	21.1	.1	21.2	.2
1965[2].....	22.2	20.3	-1.9	19.9	-2.3

[1] Equation (9): $\text{Log } P = 1.966 - 0.082\ T - 0.005\ (\Delta C)$
[2] Based on preliminary data available in January 1966.

Sufficient Conditions for Exact Aggregation in Linear Programming Models

By Thomas A. Miller

MANY GAPS exist in the economic analysis of broad problems of agricultural policy. One is the lack of a reliable method by which economists can generalize from analysis of individual farms. Current research methodology often includes scaling up the linear programming solution of a "representative" farm to generate information about the aggregate production behavior of the group or set of individual farms it represents. But this approach has a weakness. If the individual farms in the group do not respond alike to changes in economic stimuli, the estimates of aggregate output for the group will be biased. This article develops the conditions of similarity among individual farms which, if met, are sufficient to permit grouping farms so that a representative farm may be used to estimate the aggregate behavior of each group without bias.

The basic problem is how to obtain estimates of the total output of a given set of farms under various assumptions. One possible procedure would be to determine the optimum organization (and output) from every individual farm in the set and to sum them into the desired aggregate estimate. Although this procedure would result in a bias-free estimate, the limited resources available for study usually make it impractical. Alternative procedures involving greater abstractions become necessary to make the problem computationally feasible.

An often used alternative procedure is to define a "representative farm" within the set and to determine the optimum organization for this farm by linear programming techniques. The output of the set as a whole is then estimated by multiplying the solution of the representative farm by a weighting factor defined as the number of farms in the set. Since the resources of the representative farm are usually defined as the sum of all resources in the set divided by the total number of farms, a parallel method is to consider the total set and its resources as the representative farm and to determine the optimum solution for the entire set directly. These two procedures yield identical output estimates.

Inherent in these abstractions is the possibility of aggregation bias.[1] This is said to exist when the sum of the solutions for each of the individual farms in the set does not equal the estimate obtained by determining the optimum solution to the entire set directly (or the total obtained by weighting the solution for the representative farm).

The Aggregation Problem

At this point, it is desirable to make a more rigorous definition of aggregation bias and at the same time develop a notation for the discussion which follows. Consider the linear programming model representing the gth farm of the set of n farms, which is the problem of selecting a vector of production levels, X_g, such that profit is a maximum, resource limits are respected, and no production levels are negative. In the usual mathematical notation we solve for X_g such that

$$(1) \qquad \pi_g = Z_g X_g$$

is a maximum subject to

$$(2) \qquad B_g X_g = C_g$$

and

$$X_g \geq 0$$

[1] Lee M. Day, "Use of Representative Firms in Studies of Interregional Competition and Production Response," Jour. Farm Econ. 45:1438-1445, Dec. 1963.

where π_g = total net returns to the gth farm,

Z_g = the 1 by m vector of activity net returns for the gth farm,

X_g = the m by 1 vector of the activity levels to be chosen by the gth farm,

B_g = the k by m matrix of input-output coefficients for the gth farm, and

C_g = the k by 1 vector of available resources of the gth farm.

This is standard linear programming form with the necessary slack vectors included to reach equality of relations in equation (2).

If the optimum solutions are obtained for all n farms and totaled, the desired solution for the aggregate set of n farms becomes $\sum_{g=1}^{n} X_g$. This is the procedure referred to in the second paragraph of the introduction and the result is an estimate free of aggregation bias. Hence, it becomes a logical standard against which all other procedures may be judged.

As mentioned earlier, the alternatives often used are (1) to sum the total resources over all farms and to determine the optimum solution for the aggregate as a whole or (2) to weight results obtained for a representative farm. Since these procedures yield equivalent results, either one may be used in discussing the aggregation problem with no loss in generality. Choosing the former, the more abstract alternative then may be expressed in one problem of selecting a vector of aggregate area production levels, X*, such that

(3) $$\pi^* = Z^*X^*$$

is a maximum subject to

(4) $$B^*X^* = C^*$$

and

$$X^* \geq 0$$

The starred symbols represent the entire set of farms where the g subscripts represented the individual farms. The dimensions of the matrices are the same in both cases. Since the individual farm resources are summed to obtain the resources of the aggregate set,

$$C^* = \sum_{g=1}^{n} C_g.$$

Now we may define exact aggregation as the situation in which the levels of the various activities in the second formulation are exactly the same as that obtained by programming each farm separately and summing, that is

$$X^* = \sum_{g=1}^{n} X_g.$$

Conversely, aggregation bias is defined as the situation in which

$$X^* \neq \sum_{g=1}^{n} X_g.$$

The central question now becomes, given the set of n farms and the problem specified above, what conditions among the set of farms are sufficient to assure exact aggregation?

A Recent Contribution to Bias-Free Aggregation

In an article on aggregating linear programming models, Richard Day defines sufficient conditions for exact aggregation as the requirement of "proportional heterogeneity."[2] The conditions are that

(5) $$B_1 = B_2 = ..., = B_n = B^*$$

(6) $$Z_g = \gamma_g Z^*$$

where γ_g is a scalar greater than zero for all g and

(7) $$C_g = \lambda_g C^*$$

[2] Richard H. Day, "On Aggregating Linear Programming Models of Production," Jour. Farm Econ. 45: 797-813, Nov. 1963.

where λ_g, a scalar greater than zero and less than one for all g, represents the proportion of the sets' resources that the gth firm possesses. Condition (5) is that all firms must have identical matrices of input-output coefficients; condition (6) is that firms have only proportional variation in net return expectations; and condition (7) is that firms have only proportional variations in constraint vectors.

Day presents proof of the sufficiency of these conditions through the duality theorem of linear programming. In addition to fulfilling the previously defined requirements of exact aggregation, he notes that the condition

$$R^* = \frac{1}{n} \sum_{g=1}^{n} R_g$$

would also be achieved in a set of firms conforming to equations (5), (6), and (7) where R^* is the "average marginal net revenue productivities" of the resources in the set and the R_g are the vectors of marginal net revenue productivities of resources of the individual firms.[3]

Day has an excellent discussion of the implications of the conditions of proportional heterogeneity from an operational standpoint. The interested reader is urged to refer to his article. His comments concerning conditions (5) and (6) are particularly detailed. In this area I find that many current research projects in agricultural economics are based upon the assumption of a given and specified level of management and hence identical input-output matrices for large groups of firms. The assumption of proportionality in the vectors of expected net returns is likewise easily met. In fact, an often used procedure is merely to assume all firms have the same net return expectations.

This article is primarily concerned with the restriction posed by condition (7), which allows only the variation in resources among firms that are usually expressed as differences of scale of operation. If two firms differ in one resource by a certain ratio, they must differ in all other resources by that ratio. This condition appears very restrictive from the operational standpoint. For example, in agriculture, there are nearly as many different cropland resource situations as there are farms, while, on the other hand, a majority of these same farms may have labor supplied by only one operator. The implications should be obvious to researchers working in this area, and it does not appear desirable to go into them in any great detail. Instead, I have developed less binding sufficient conditions for exact aggregation.

[3] These values represent the solution of the dual linear programming problem.

Less Binding Requirements

The first step is to define the less binding requirements both intuitively and then somewhat more rigorously. A theorem and proof of the sufficiency of these requirements then follow.

An intuitive idea of the relaxed requirements is gained by considering the optimum solutions of a set of individual farms as determined by linear programming. Assume the set of farms under consideration is similar to the extent necessary for all individual optimum solutions to include identical sets of activities. Such a set of individual farms may then vary in both resource and net return vectors, so long as this variation is not great enough to cause a change in the set of optimum activities common to all farms in the group. The variation in net return and resource vectors among farms will, of course, cause differences among farms in optimum activity levels. The important thing is that the identity of the activities in the optimum solutions must be the same for all farms. Farms meeting this requirement will be defined as having qualitatively homogeneous output vectors.

To make a more rigorous specification of the new conditions, consider the optimum solution of each individual farm. The optimum solution for the gth farm may be expressed as a column vector

$$X_g = \begin{bmatrix} X_{1g} \\ X_{2g} \\ \cdot \\ \cdot \\ \cdot \\ X_{mg} \end{bmatrix}$$

Previously, m was defined as equal to the number of production processes considered by the farm plus the number of slack vectors necessary to permit nonuse of resources and k was defined as the number of resources or constraints. Observe that $m > k$ for this formulation since k is also the number of required slack vectors that are included in m to achieve equality in the restraints. For each optimum solution, X_g is made up of at most k activities that are greater than zero and at least m minus k activities that are equal to zero.[4]

Now consider a set of farms which have qualitatively homogeneous output vectors. For each of these we could express a streamlined output vector as

$$X'_g = \begin{bmatrix} X'_{1g} \\ X'_{2g} \\ \cdot \\ \cdot \\ \cdot \\ X'_{kg} \end{bmatrix}$$

by omitting the m minus k activities which are common to each and equal to zero. We note now that the X'_g (streamlined output vectors) for all farms will all consist of the same k basic activities. All such farms will have the same resources limiting, the same resources in disposal, and the same real processes in their final solution vectors. This leads to the following theorem.

Theorem. Sufficient conditions for exact aggregation are (1) that all farms have identical coefficient matrices, that is, that $B^* = B_g$ for all g, and (2) that all farms have qualitatively homogeneous output vectors.

Proof. For farms meeting conditions of the theorem, the original linear programming problem may be narrowed to the more trivial problem of solving a set of k equations in k unknowns

$$B'^{*}X'_g = C_g \tag{8}$$

where $B'_g = B'^{*}$ is the k by k part of the coefficient matrix corresponding to the k activities in X'_g. Equation (8) is then equivalent to equation (2) with the unused activities of the coefficient matrix omitted and the zero elements of X_g omitted. This is no more than saying that if the identity of the final basis activities is known in advance, the linear programming problem may be solved simply as a set of simultaneous equations.

Similarly, the solution to the aggregate farm may be determined from the relation

$$B'^{*}X'^{*} = C^{*} \tag{9}$$

which is developed in a similar fashion from equation (4).

Summing equation (8) over all n farms gives

$$B'^{*} \sum_{g=1}^{n} X'_g = \sum_{g=1}^{n} C_g$$

Since $\sum_{g=1}^{n} C_g = C^*$ by definition, it is obvious from equations (8) and (9) that $X'^{*} = \sum_{g=1}^{n} X'_g$. All that remains is to include the m minus k zero level elements to both vectors to complete the proof that $X^* = \sum_{g=1}^{n} X_g$. The conditions of the theorem are hence sufficient conditions for exact aggregation.[5]

[4] These k activities are often called the basic variables in the literature, while the remaining activities are called nonbasic variables. The theorem generally developed is that an optimum solution involves at most k unknowns at nonzero values (where k equals the number of equations). For example, see: R. D. Dorfman, P. A. Samuelson, and R. M. Solow, "Linear Programming and Economic Analyses," New York, McGraw-Hill Book Company, Inc., 1958, Theorem 2, p. 75.

[5] The conditions of the theorem are general in respect to the price or revenue vectors used; hence, the theorem covers variable-price programming. This is because the consideration of different prices has the effect of further restricting the groups of farms that have qualitatively homogeneous output vectors. To have exact aggregation under varying sets of prices, all farms in the group must merely meet the conditions of the theorem for every set of prices considered. In other words, the farms must all have qualitatively homogeneous output vectors for the first set of prices, have a set of possibly different but again qualitatively homogeneous output vectors for the second set of prices, and so on for all price ratios considered.

A parallel argument could be developed for aggregation of the dual solutions over the same set of n farms to obtain the "average marginal net revenue productivities" of the resources. Under conditions stated in the theorem, if $C^* = \sum_{g=1}^{n} C_g$, then $X^* = \sum_{g=1}^{n} X_g$. Likewise, for the dual solutions the same argument may be developed to show that if $Z^* = \frac{1}{n} \sum_{g=1}^{n} Z_g$, then $R^* = \frac{1}{n} \sum_{g=1}^{n} R_g$ where Z_g are (as defined earlier) the vectors of expected net returns per unit of the respective activities and the R_g are the vectors of desired "marginal net revenue productivities" of the resources in the optimum solutions of each individual farm. Hence, the conditions of the theorem also appear to be sufficient conditions for exact aggregation of the "marginal net revenue productivities" of the resources of the individual farms into the "average marginal net revenue productivities" of all resources in the aggregate.

Implications of New Conditions

The problem of aggregation bias is not trivial to research workers in agricultural economics. Large amounts of money are being allocated to projects which are utilizing the representative farm linear programming concept in developing aggregate area production estimates and area supply functions. One example is the NC-54 regional project, "Supply Response and Adjustments for Hog and Beef Cattle Production," in which 13 Corn Belt States are cooperating. Iowa's contribution to this project involves the use of 63 farms to represent all commercial farms in the State. Results from these 63 farms are used to develop State supply functions for hogs and beef cattle. At later stages, similar supply functions from all cooperating States will be combined.[6]

[6] For results of a recently completed study using similar methodology, see: W. B. Sundquist, et al., "Equilibrium Analysis of Income-Improving Adjustments on Farms in the Lake States Dairy Region, 1965," Minn. Agr. Expt. Sta. Tech. Bul. 246, Oct. 1963.

The question of how sufficient conditions for exact aggregation affect such current research studies is certainly important. The conditions developed in this paper are substantially less binding from an operational standpoint than the original ones developed by Day (see footnote 2). Some range of different resource situations and net return expectations can now be combined without incurring aggregation bias. Moreover, there is no restriction on the type of variation that may occur between farms, so long as all of the individual farms in the set have solutions made up of the same activities.

On the other hand, the new conditions are defined as a requirement of the solutions to the individual farms rather than a requirement of the farms themselves. Therefore, they provide less than an ideal solution to the problem of delineating representative farms. It may be difficult to anticipate the solutions of various individual farms with the accuracy necessary to stratify them into the separate classes required to avoid aggregation bias. Considerable prestratification analysis may be necessary. Nevertheless, the theorem still provides the researcher with a definite idea of the objective of the stratification. This is to delineate sets of individual farms in such a way that all farms within a respective set will meet the conditions of (1) identical input-output matrices, and (2) qualitatively homogeneous output vectors.

In addition, it may be desirable to answer such questions as: Given a representative farm, to what extremes may its coverage be extended? This question may arise after some preliminary linear programming work is done with the basic data, and a general idea is obtained of the types of optimum solutions involved and the effect of variance of different resources upon them. The answer can be obtained by parametric programming on the resource vector of the representative farm. The results will give the ranges of individual farm resource vectors which may be included in the set represented by that particular representative farm, without creating aggregation bias. This assumes, of course, that the appropriate adjustment will be made in the representative farm's resource vector.

Little can be added to these general statements. The problem of specifically how many representative farms are required to avoid

aggregation bias in a given instance is left unsolved. The problem of how rapidly aggregation bias accumulates as we move away from the sufficient conditions stated in the theorem is also unsolved. Both of these are essentially empirical questions which must be answered by empirical means for each individual research problem.

Exact Aggregation--A Discussion of Miller's Theorem

By John E. Lee, Jr.

TOM MILLER, in an article in this journal,[1] has shown that the optimal responses of different farms to a given set of relative product prices will be proportional if two conditions are met. The conditions are that the farms have homogeneous activity vectors and that the same activities appear in the linear programming solution vector for each farm.

Miller's paper and this discussion of it are concerned with the aggregation problem, which is essentially one of grouping farms that respond alike in linear programming models of agricultural supply. Miller starts by observing that levels of management and practices are assumed given or are specified in most research projects so the input-output matrices for large groups of farms are identical. He further observes that all farm managers can be assumed to behave in a consistent, objective manner; thus, an acceptable procedure is to assume all farmers have the same net return expectations. These two observations allow Miller to focus his concern on Richard Day's condition of proportionality of resource vectors among all farms in the aggregate.[2] Miller sets forth the following theorem:

> "Sufficient conditions for exact aggregation are (1) that all farms have identical coefficient matrices, that is, that $B^* = B_g$ for all g, and (2) that all farms have qualitatively homogeneous output vectors,"

where the asterisk (*) denotes the aggregate set of farms and the subscript g represents the individual farms. "Qualitatively homogeneous output vectors" means that all farms in the aggregate have the same activities included in the solution vector.

Several points can be made about Miller's aggregation theorem. One weakness is that the groups of farms having qualitatively homogeneous output vectors are unique for each set of relative product prices. This is because, ceteris paribus, the nonzero activities in the solution vector depend on relative activity net returns. These, in turn, depend on relative product prices. For each additional set of prices considered, all farms have to be reprogrammed to determine which farms are common to a given group over the whole range of prices. This does not invalidate Miller's theorem. It does imply that in a model designed to estimate supply response to wide price changes, a burdensome amount of computation may be required.

Richard Day's proportionality conditions are general with respect to price changes; that is, the same farms lend themselves to exact aggregation at all price combinations. (However, one could point out that Day's farm groupings are not general if one considers, say, the disproportionate effects on the resource structure of farms resulting from changes in Government allotment programs.) A group of farms aggregated under Day's more restrictive proportionality conditions are a subset of a group aggregated under Miller's less restrictive conditions for a specified set of product prices. As the product price ratios are varied over an infinite range, Miller's sets of farms approach but do not reduce to Day's subsets. The import of this will be pointed out later.

Another shortcoming of Miller's aggregation theorem centers around its practical applicability. The fact that his conditions are defined as a requirement of the solutions to the individual farm problems, rather than a requirement of the farms themselves, provides a less than ideal approach to the problem of delineating representative farms. Miller recognizes this. Day's conditions could be used to group farms simply by observing the farm characteristics.

Miller's work, as it stands, represents progress. In a fairly homogeneous farming area, large groups of farmers employ similar

[1] Thomas A. Miller, "Sufficient Conditions for Exact Aggregation in Linear Programming Models," this journal, this issue, p. 52.

[2] Richard H. Day, "On Aggregating Linear Programming Models of Production," Jour. Farm Econ., Vol. 45, Nov. 1963, pp. 797-813.

production practices, and view essentially the same alternatives. Thus, they have similar coefficient matrices and similar sets of activities in their "subjective solution vectors." In addition, one may be concerned with supply response to a relatively narrow range of price ratios such that the subset of farms contained in the unbiased aggregate could be easily determined for that range of prices.

However, these practical observations may not be the most valuable results of Miller's work. Miller hinted at but did not exploit an extension of his analysis, which could potentially lead to translation of the farm solution vector conditions into observable characteristics of the farms themselves. This potential is revealed in the dual to Miller's primal problem.

In the primal problem, Miller proved that for a group of farms having qualitatively homogeneous solution vectors and identical B matrices, if

$$C^* = \sum_{g=1}^{n} C_g, \text{ then } X^* = \sum_{g=1}^{n} X_g.$$

For the dual solutions, Miller observes that a parallel argument could be developed to show that if

$$Z^* = \frac{1}{n} \sum_{g=1}^{n} Z_g, \text{ then } r^* = \frac{1}{n} \sum_{g=1}^{n} r_g.$$

Note that the marginal revenue products of the n farms are not weighted by the relative share of aggregate resources belonging to each farm (as was the case in Day's paper). The shadow prices of the individual farms are simply added and divided by n, and they turn out to be exactly the same as the shadow prices of the aggregate problem. It follows then that if the net returns expectations are identical for all farms in the group,

$$Z_g = \frac{1}{n} \sum_{g=1}^{n} Z_g.$$

Thus

$$Z^* = \frac{1}{n} \sum_{g=1}^{n} Z_g = Z_g$$

for any and all farms. From this, it is obvious that

$$r^* = \frac{1}{n} \sum_{g=1}^{n} r_g = r_g$$

that is, the marginal revenue products are the same for all farms and are constant over the range of resource ratios represented by the aggregated farms. It now becomes clear that the observed ranges of resource ratios represented can be used as criteria for grouping farms on the basis of observable characteristics. In effect we have developed a new aggregation theorem which can be stated as follows:

> "Sufficient conditions for exact aggregation are (1) that all farms have identical coefficient matrices, (2) that all farms have the same net returns expectations, and (3) that the range of resource ratios be such that the dual solution vector is the same for all farms."

This is simply the dual counterpart of Miller's theorem. It would delineate sets of farms identical to those delineated by the original theorem. However, it may be more useful since it lends itself to interpretation in terms of observable characteristics. The link between theorem and application is the empirical task of determining the exact ranges of resource ratios over which the marginal revenue product is constant.

The potential of this approach may be demonstrated graphically. Suppose there exists a group of farms each possessing some combination of two resources C and L, each viewing the same three production processes, A_1, A_2, and A_3, with technical coefficients common to all farms, and each having identical net returns expectations. The situation is depicted in figure 1.[3]

[3] The graphic exposition was first suggested in private correspondence to Lee M. Day by John Stovall of the University of Kentucky. Stovall reported the attempts of a graduate student at the University of Kentucky to group farms on which capital and labor resources were fixed and all other resources variable. The student held capital constant and obtained primal and dual programming solutions with varying amounts of labor. He then determined the points at which the marginal value product (MVP) of labor changed and drew lines from the origin through these points. He called these lines "MVP boundaries" but did not indicate awareness that these boundaries were in fact activity vectors or that they represented the resource ratios at which the optimum activity mix changed.

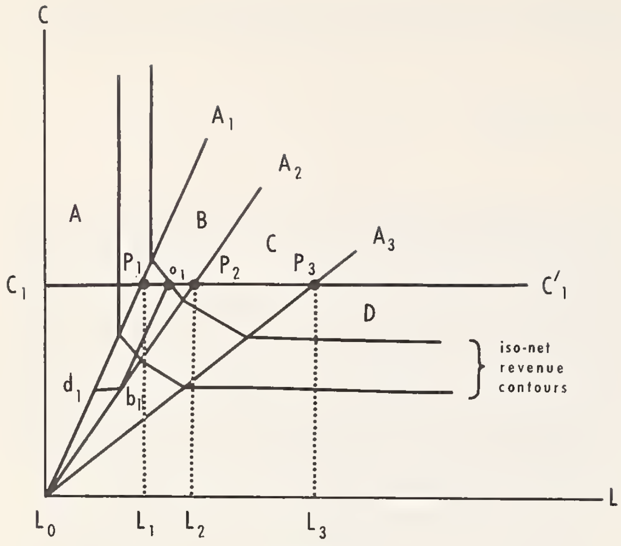

Figure 1

All possible resource ratios are depicted by points on the horizontal bar C_1 C_1' derived by adding varying amounts of resource L to a fixed amount (C_1) of resource C. The net returns expectations for an initial set of product prices (or input prices) create a field of iso-revenue curves exemplified by the solid iso-net revenue curves shown.

Given the situation described above and portrayed in figure 1, the output and net revenue for farms with C_1 of resource C and none of resource L will be zero. As the level of resource L increases in small increments from L_0 to L_1, L remains the limiting resource and net revenue increases in proportion to increases in L.[4] In other words, as resource L is increased from L_0 to L_1, net revenue is maximized by moving up the iso-revenue field along the A_1 activity vector (since the A_1 vector represents the most efficient utilization of resources as long as resource L is limiting).

Since net revenue changes in proportion to changes in the limiting resource, L, between L_0 and L_1, the marginal value product (shadow price) of L is constant over the same range. Under the conditions of the "dual aggregation theorem" and under the assumptions applied to figure 1, all farms with resources C and L in ratios ranging between C_1/L_0 and C_1/L_1 can be aggregated without bias so that

$$X^* = \sum_{g=1}^{n} X_g.$$

With resource combination $C_1 L_1$ (denoted by point P_1), both resources are exactly used by activity vector A_1. With further increases in resource L (beyond L_1) both resources C and L are limiting. However, the full amount of both resources can be utilized and net revenue maximized by combinations of activities A_1 and A_2 (for example, L_0 b_1 of A_2 and b_1 a_1 = d_1 P_1 of A_1 in figure 1). The locus of resource combinations, P_1 P_2, is also the path of net revenue expansion as resource L is increased. This expansion path intersects the iso-net revenue field at constant angles (i.e., as L increases, the net revenue from A_2 substitutes for net revenue from A_1 at constant rates). Thus, between L_1 and L_2, the marginal revenue product of L is constant and the conditions of the dual to Miller's theorem are again met. Note that the MVP of L between L_1 and L_2, while constant, is less than the constant MVP of L between L_0 and L_1. The reason is that as L is increased it becomes less scarce relative to resource C. This is reflected in the flatter slope of the iso-revenue curve. Obviously, farms with resource ratios between C_1/L_1 and C_1/L_2 can be aggregated without bias.

As resource L is increased from L_2 to L_3, its MVP is again constant though lower than previously. Farms with resource ratios between C_1/L_2 and C_1/L_3 meet the conditions for exact aggregation. Beyond L_3 amounts of resource L, C becomes the only limiting resource; A_3 is the only activity in the solution and the MVP of L is constant at zero. Thus, all farms with resource ratios of C_1/L_3 or less can be aggregated without bias.

With resource C fixed at C_1, the line L_0 P_1 C_1' represents the maximum efficiency net revenue expansion path as L is increased from L_0 to infinity. The angle at which this path cuts the field of iso-net revenue curves determines the marginal revenue product (shadow price) of L.

Thus, in figure 1, at the set of prices reflected in the iso-revenue contours, the number of

[4] Assuming, of course, no internal or external economies or diseconomies of scale.

groups of farms needed to eliminate aggregation bias is four. In effect, the activity vectors together with the axis of figure 1 represent MVP "borders." All farms with combinations of C and L falling between two adjacent "borders" have the same shadow prices in the dual, have the same activities in the primal solution vector, and can be aggregated without bias.

If the activities A_1, A_2, and A_3 are the only alternatives available to the farms, the four groups of farms, A, B, C, and D, represent the maximum number of groupings needed for zero-bias aggregation purposes, regardless of the relative product or input prices. For a specific set of prices, a smaller number of groups may suffice for zero-bias but will not be general.

The preceding exposition can be used to demonstrate that Miller's sufficient conditions for exact aggregation are indeed less restrictive than Day's. In figure 1, every point on the locus of resource ratios, $C_1 C_1'$, represents a separate group of farms under Day's condition that each farm in a group have resources in the same proportion. One of Miller's groups contains all such points--and all of Day's groups--that lie between two MVP "boundaries." Miller's groups (of farms which can be aggregated without bias) reduce to Day's groups only when the number of alternative activities is infinite.

It is apparent from the preceding discussion that the key to determination of the resource ratios relevant to bias-free grouping of farms is the relationship between the ratios in which resources are required by alternative activities and the ratios in which resources are available to farms. In the two-dimensional example, the marginal value product borders are determined from the technical coefficients themselves without having to solve the linear program. A generalization of the grouping procedure illustrated in that example is now being developed for extension to multiproduct-multiresource farm populations.

In summary, Miller's theorem does not represent a final resolution of the problem of aggregation bias in linear programming models. However, an extension of the dual to that theorem indicates the range over which resource ratios may vary without introducing bias. The ideas presented, when combined with those of Day, Stovall, and others, provide the pieces from which a general, practical aggregation procedure will eventually be developed.

Book Reviews

The Economic Demand for Irrigated Acreage

By Vernon W. Ruttan. The Johns Hopkins Press, Baltimore. 139 pages. 1964. $4.

THIS LITTLE BOOK is an analytical piece loaded with information and appraisals of the nature and prospective demand for irrigated land in the United States. Although the author makes some policy judgments that spring from his research, I think the major theme of the book is methodological. It sets up some of the many necessary economic dimensions to be considered in determining relative profitability of public and private investment in irrigation development.

"Economic Demand for Irrigated Acreage" and its fairly extensive footnotes bring the reader to date on recent developments in projection techniques. Many of these developments are related to the recent expansion in the availability of rapid computers.

Since the book is methodological, I would like to move directly to the innards of the story, skipping over the introductory chapter and a good historical perspective on resource utilization in agriculture. Ruttan's analytical framework at first glance seems entirely too simple, but you will find it ingenious and useful. It incorporates some important economic dimensions and remains simple enough for the wide regional applications presented in the study.

The "productivity model" consists of a Cobb-Douglas production function from which a "marginal value product" is derived for irrigated land and current operating expenses. The production function expresses value of farm products sold (X) as a function of acres of irrigated land (L) and current "output-increasing" operating expenses (E):

$$(1) \qquad X = A L^a E^e$$

From this function, the marginal value product for irrigated land, our primary interest is:

$$(2) \qquad M_a = \left(\frac{X}{L}\right) a$$

The author then determines national output (X) essentially on the basis of demand--output is determined by population and income growth, assuming an income elasticity of 0.15. Regional output is estimated from national output on the basis of past trends. To illustrate the general framework, the author first makes an independent determination of national output and from this determines output for the region. With regional output, the demand for irrigated land (L) is determined in (2) given values for (a) and (M_a) estimated from the statistical fit of the regional production function. One other addition to the model is an identity equating the computed marginal value product (M_a) to the cost of bringing an acre of irrigated land or other input into use (M_c). This permits the easy substitution of the marginal cost (M_c) for the marginal value product (M_a) relating the demand for irrigated land (L) to alternative costs.

A slight further complication of the above framework gives the equilibrium model. In addition to regional production functions (1), the equilibrium model contains marginal value product functions (2) for irrigated land (M_a) and for operating expenses (M_e) as well as two identities equating calculated marginal value products to budgeted input costs. This model, which simultaneously determines output and inputs, indicates acreage that would result by profit maximization, assuming constant input and product prices as well as no change in technology. The equilibrium model was used primarily as a frame of reference rather than as a basis for projections for specific future dates.

Armed with statistically fitted regional production functions and his analytical framework, the author computes marginal value productivity estimates for major inputs. Current irrigation costs as well as the much higher projected "full" costs of irrigation were also estimated for each region. Estimates were then made for 1954 and projected for 1980 assuming current costs and estimated full irrigation

costs. No attempt will be made to summarize all these projections and their policy implications (chapter 6). In general, results based on the model were better for Western regions than for Eastern regions where there is little irrigation.

The book has much to recommend it, but my review would be less than complete without raising some questions and pointing out some limitations. I am sure that most, if not all, these questions are well known to Ruttan.

The author uses a rather unsophisticated demand relationship as a basis for projecting national output, and the link from the national to the regional level is a weak one. Export demand, currently a big uncertainty, is not adequately provided for in the author's framework. There is no explicit handling of the important effects of price on demand and on output in agriculture. Recent recursive frameworks in use in the Department of Agriculture estimate an equilibrium price or show the impact of several price and cost alternatives.

Possibly one of the biggest shortcomings of the model for projections work is that it does not reflect technology. The use of budgeted cost levels, which imply a completely elastic supply response for inputs, also limits the use of the model.

Statistically fitted regional production functions show considerable variability in factor productivity for alternative formulations and for different regions. These variations plus some relatively flimsy regional data give cause for caution in drawing precise policy conclusions. But this the author knows well.

The methodology, background information, and projections in the book are well worth your time and effort. The book will help to improve long-run decisions--possibly not with specific answers, but with an improved analytical tool.

Rex F. Daly

Roots of the Farm Problem

By Earl O. Heady, Edwin O. Haroldsen, Leo V. Mayer, and Luther G. Tweeten. The Iowa State University Press, Ames. 224 pages. 1965. $4.95.

A POPULARIZATION of earlier research as presented in "Resource Demand and Structure of the Agricultural Industry," Ames, 1963, this book contains a few additional interpretations and results of some later research. The central thesis is that the farm problem in the United States--production in excess of market demands (the symptom)--arises from rapid substitution of capital inputs for labor (the cause).

Agricultural output has increased continuously since 1870, the authors say, but it was the mid-1930's before output rose faster than inputs. The productivity of inputs increased greatly after 1940. Capital input per unit of labor input more than doubled between 1940 and 1960. Total agricultural output gained roughly 60 percent in that period, with practically no change in the aggregate of farm inputs. Annual use of farm labor decreased from 20 billion to 10 billion man-hours from 1940 to 1960; use of cropland for crops also declined. But sharp increases occurred in use of capital inputs, including machinery and power, fertilizer, lime, other agricultural chemicals, and purchased seed, feed, and livestock. Farming practices also were greatly improved.

With improved inputs, output increased faster than demand and prices fell, the authors state. As a result, labor returns declined relative to nonfarm incomes, and labor was encouraged to migrate from the farm.

But what has been the motivation for changes toward greater use of capital inputs? It comes primarily from technological advances and competitive pressures from other sectors of the economy, the authors say. Farm wage rates, while low in absolute terms, have become increasingly high in relation to farm prices and to costs of other inputs. The vast accumulation of social overhead in the United States was a prerequisite to technological advances and the wide adoption of capital inputs. The authors mention education and both public and private contributions to technology. They also mention the goals and values of farmers and incentives offered by the profit motive. They might have said more about other elements of social overhead, such as access to markets provided by waterways and highways; freedom from religious taboos; and favorable systems of inheritance.

Attention is given to opportunities for substituting inputs, and to demand for the different kinds of inputs, including estimates of demand

elasticities. Farmland prices, and changes in number and incomes of farms, by farm size, also are discussed. On the subject of commodity supply and farm income adjustments, the authors indicate that output response to price change is relatively slight in the short run, and that reducing price supports cannot be depended on to improve the farm income situation. Output appears to be more responsive in the long run, that is, in a period of more than 20 years. In the short run, they report, a 10 percent decline in farm prices might be associated with a 1 percent decline in production; over the longer period production might decline by 5 to 7 percent.

Seventeen pages are devoted to a discussion of different schemes for adjusting farm output to demand. This section suffers somewhat from brevity and an abundance of assumptions. The authors reach the conclusion that "net social welfare may not, in fact, be maximized by free markets."

"A Picture of Agriculture in 1980" is a fitting conclusion to any contemporary book in agricultural economics. Total agricultural output requirements projected for 1980 range from 35 to 50 percent above the 1960 value. Input requirements are based on straight-line extensions of the 1950-60 trends in input productivity, corrected for weather. These projection techniques are admittedly naive. The results in terms of total output are somewhat lower than those given by Daly and Egbert in the January 1966 issue of this journal ("A Look Ahead for Food and Agriculture"). However, no set of projections 15 to 20 years ahead is likely to be highly accurate. The projections of inputs, moreover, even if only roughly approximate, are useful in farm policy formulation and for industries supplying goods and services to farmers.

Much in the book is not new. But the authors present a highly readable account of the causes and consequences of farm troubles, and of the economic forces that have increased the use of fertilizer and chemicals, farm machinery, and operating inputs, while decreasing the need for farm labor.

Robert M. Walsh

Index Numbers, Theory and Application

By Walter R. Crowe. McDonald and Evans, Ltd., London. 368 pages. 1965. $6.95.

THIS VOLUME on "Index Numbers, Theory and Application" is light on Theory and heavy (relatively at least) on Applications. Theory is indeed touched upon, but after the "touching" the author almost invariably refers the reader to other writers. This is true to such a degree that the term "Theory" deserves no place in the title.

Perhaps the philosophical level of the book is best characterized by a paragraph occurring on page 84, near the end of the cursory chapter on "Tests for Perfection." The paragraph is:

> "Have we now discovered the perfect answer? Whatever the mathematical virtues of a formula the main considerations determining the choice are practical--ease of calculation and all around reliability."

And not much has been offered by the author as a means for determining reliability.

The book is divided into two parts. Part I is entitled "The Theory of Index Numbers" and consists of a once-over very lightly of such conventional subjects as "which average," "what base," "relatives or aggregates," and "weighting," that usually receive more comprehensive treatment in a good academic text. One section in the 10-page chapter on "Various Adjustments" is "Time Series Analysis," a subject that can scarcely be covered in a whole chapter, let alone in a section of a short chapter. Perhaps the best chapter in part I is that on "The Development of Index Numbers." Although the chapter does not delve deeply into the real problems of index numbers it traces the development of the idea of an index, mentions some of the contributions of various writers, and introduces the reader to some of the "Greats" in the literature. However, the work of Wesley C. Mitchell is barely mentioned (and that not in the history chapter).

The chapter entitled "Tests for Perfection" names and illustrates the Time and Factor Reversal Tests and the Circular Test. It touches

only superficially on their implications, and fails completely to provide the student with an understanding of their central position in Index Number Theory. It seems to miss the point that the reversal tests have for their purpose the testing of formulas to determine whether they are biased, and the nature of the bias. The discussion of the Circular Test is completely inadequate and at best can only leave the student confused.

Part I includes a quick presentation of sampling (chapter 11) and of correlation (chapter 12). Chapter 11, however, does not touch on how the theory of sampling should be applied to price collection or to index numbers nor does chapter 12 explain how correlation between prices and quantities affects formula bias. The matters of formula bias and weight bias are mentioned so casually and tangentially that they might nearly as well have been omitted entirely.

Formula (4), page 48, should read

$$\log p_{01} = \frac{\Sigma \log \frac{p_1}{p_0}}{n}$$

or better

$$\log p_{01} = \frac{\Sigma \log p_1 - \Sigma \log p_0}{n}$$

The line following the table at the top of page 49 should read

$$\log p_{01} = 1/3 \left[\Sigma \log p_1 - \Sigma \log p_0 \right]$$

$$= \frac{0.0669}{3} = 0.0223$$

whence $p_{01} = 105.3$.

Part II is devoted to a description of a number of the leading indexes published in Britain and the United States. The discussion is necessarily synoptic and the reader desiring specific information will need to consult original sources. This is, however, probably the most useful portion of the book, providing as it does a bird's-eye view of available indexes. The reader desiring a general view of available indexes will find this section of the book a helpful compilation.

The preface of the book opens with the statement, "This book is intended to assist students in first-year university courses, in technical colleges, those studying at home, and others interested in statistics by explaining the basic theories on which Index Numbers are calculated and by giving a comprehensive treatment of the applications of Indexes published or available in Great Britain plus a selection of important American ones." A little later is the statement, "It is hoped that it will be used in addition to the standard college textbook."

This reviewer hopes that a standard college text would be used in addition to the volume under review.

A moderately extensive bibliography is included. Readers in the United States, however, will be surprised to see publication of the Journal of Farm Economics credited to the U.S. Department of Agriculture rather than to the American Farm Economic Association.

B. Ralph Stauber

National Growth and Economic Change in the Upper Midwest

By James M. Henderson and Anne O. Krueger. The University of Minnesota Press, Minneapolis. 231 pages. 1965. $7.50.

THE UPPER MIDWEST Economic Study (UMES) was begun in 1960 as a joint project of the University of Minnesota and the Upper Midwest Research and Development Council. The three major objectives of the research were: (1) provision of information and analysis that would be of aid for programs to stimulate Upper Midwest regional growth; (2) advancement of methods for regional economic analysis; and (3) analysis of interactions between general economic development and urban planning within the region. The work has been reported in a series of 9 "study papers," 8 "urban reports," and 11 "technical papers," as well as the present final general report.

In the present report emphasis is placed upon future development possibilities and policy

action alternatives of the Upper Midwest. This region coincides with the Ninth Federal Reserve District, which includes Montana, North and South Dakota, Minnesota, Michigan's Upper Peninsula, and 26 counties in northwestern Wisconsin. The first three chapters cover the region's 1950-60 economic structure and growth in terms of trade, employment, income, population, and migration, the 1960-75 projections of regional economic activity, and a general consideration of regional policy formulation and action. Chapters 4-11 are more detailed discussions of the current problems, future prospects, and action alternatives of natural and human resources, manufacturing, transportation, services, and Government. Chapter 12 discusses monitoring future development of the Upper Midwest in terms of data and methods of analysis, while the appendixes discuss in greater depth the methods underlying the projections and the need for and use of associated economic data.

The UMES projections, based in part on the 1976 national projections of the National Planning Association, are termed "neutral" inasmuch as they take no account of possible effects of regional policy and indicate only what may be expected if past rates of change hold in the future.

In their discussion of policy and action, the authors consider the divergence among goals--national vs. regional, increased employment vs. increased per capita income, long-run vs. short-run costs and benefits. The levels of decisions (individual, community, State, Nation) and types of actions (legislative, administrative, regulatory, promotional, informational) are also briefly but meaningfully considered.

The authors endeavor to stop short of specific policy recommendations. The indication of likely economic costs and benefits of several alternative actions (or inactions), however, make clear in several instances biases which this reviewer also holds. For example, as a means to attack the low-income problem of farms, creation of new jobs in nearby urban centers is suggested. Nonfarm jobs not only supplement farm income but, in drawing off underemployed agricultural labor, facilitate the consolidation of farms and permit higher returns for those that remain in farming. The discussion of human resources particularly in terms of investment and the quality of education makes a strong argument for the economic benefits of Federal aid to education in the Upper Midwest.

Continuing changes in productivity and variations in requirements for transportation, services, and Government activity as far as they directly relate to income are incorporated in the UMES projections. Other intersector linkages, however, receive only implicit recognition. For example, while constraints directly affecting growth of primary metal industries were considered, the indirect effects of such constraints on metal-using industries are only subjectively considered, if at all. Greater detail with respect to the interactions among industries and areas, and careful analysis of the relevance of the underlying assumptions, would have provided an even more meaningful study.

In a general report of perhaps the most comprehensive regional study to date, it is not surprising to find a few statements on method which are vague. The reader is referred in numerous instances, however, to the earlier UMES reports and papers which unfortunately were not available in all cases to the reviewer.

The present UMES report makes a major contribution to the literature on regional growth by its explicit consideration of interstate linkages and interactions. The implications for a systematized regional economic monitoring system for policy purposes are profound, and warrant the serious attention of all concerned with regional growth and development.

H. Albert Green

Forestry in Communist China

By S. D. Richardson. The Johns Hopkins Press, Baltimore. 237 pages. 1966. $6.95.

IN 1963 S. D. Richardson, Director of Research of the New Zealand Forest Service, toured Communist China. He is one of the few Western foresters to obtain a first-hand impression of conditions there in recent times. From observation and from data made available to him from many sources--including a file of translations of official Chinese news releases relating to forestry--he has made this report as realistic as possible. Repeatedly,

he calls attention to inconsistent data given to him and implies that the "lily" may have been "gilded" elsewhere.

In scope the book covers a description of (1) the country's natural vegetation, soils, and land use; (2) forestry administration and policy; (3) silvicultural practices; (4) water conservancy and erosion control procedures; (5) the economic phases of timber production, forest products manufacture, and trade; and (6) forestry education and research.

The total land area of Communist China is slightly greater than the total land area of the 50 States in the United States. Population is more than three times that of the United States. Cultivated area is 11 percent of total land area in contrast with 20 percent in the United States. Total forest area is variously estimated at 5 to 10 percent of the total land area as against 33 percent in the United States. In general the natural forests predominate in remote areas, whereas the most active industrial development is taking place in areas that lack any substantial forest resource.

Planned industrial development of the nation will require an adequate timber supply. Expansion in mining will call for an additional volume of pit props. Much-needed extension of railway lines will require large volumes of wood crossties. Production of paper and paperboard for printing and for containers for industrial products is expected to increase. Totaling these and other requirements, a substantial deficit of forest products can be anticipated after 1975 to 1980. The spiraling population growth is likely to accentuate this deficit. Prior to this date, timber from the natural forests can be expected to care for demand.

During the past 15 years Chinese statistics on reforested areas have been startling. Richardson discounts these optimistic claims for two reasons: (1) some statistics are contradictory, and (2) survival in planted stands has been low, in many instances no more than 10 percent. Apparently, the Chinese foresters expect the products of these plantations to make up a substantial portion of the prospective deficit after 1975 to 1980. The author is skeptical that the deficit can be met in this way. Intensification of forestry effort plus increased imports and increased use of wood substitutes will probably be required if planned goals are to be met. In a word, the future wood supply picture is not bright, and colossal efforts will be required to change it substantially.

The book also provides up-to-date information on forestry education and research in Communist China and has 32 pages of excellent photographs of forests and forest enterprises. It is long on observation and on general description resulting from library research. That it is short on solid facts is not surprising. In fact, it is remarkable that the author, obviously a competent observer and analyst, is able to present as complete a general picture as he does. Until Communist China decides to obtain and release more reliable statistics than it has in the past, or to admit more foreign observers, this book is likely to be widely quoted as the most authoritative and thorough study of current Chinese forestry.

Robert K. Winters

Selected Recent Research Publications in Agricultural Economics Issued by the U.S. Department of Agriculture and Cooperatively by the State Universities and Colleges[1]

Boykin, Calvin C., Douglas D. Caton, and Lynn Rader. ECONOMIC AND OPERATIONAL CHARACTERISTICS OF ARIZONA AND NEW MEXICO RANGE CATTLE RANCHES. U.S. Dept. Agr., ERS-260, 25 pp., January 1966.

Representative ranches in Arizona and New Mexico vary in size from 34 to 512 animal units, with a total investment per animal unit of from $256 to $722. Average costs and average net income per animal unit are $49 and $15, respectively. Ranches of more than 100 animal units in size are estimated to have positive net incomes.

Campbell, George W., Walter W. Pawson, and Aaron G. Nelson. SUGAR BEETS: ESTIMATED COSTS AND RETURNS, A PRELIMINARY REPORT. Ariz. Agr. Ext. Serv. and Ariz. Agr. Expt. Sta., Tucson, 12 pp., November 1965. (Econ. Res. Serv. cooperating.)

Central Arizona farmers will begin planting 20,000 acres of sugarbeets in the fall of 1966. It is estimated that sugarbeets will produce a gross income of about $278 per acre. Yields and production costs for sugarbeets in central Arizona are expected to be quite similar to those in the Imperial Valley of California.

Cooper, Maurice R., and William F. Harris. SHIPPERS' COSTS OF ASSEMBLING AND DISTRIBUTING SOUTHWESTERN COTTON, BY TYPES, MARKET TRADING AREAS, AND SALES OUTLETS, SEASON 1964-65. U.S. Dept. Agr., ERS-261 (1965), 2 pp., November 1965.

Estimates included are based largely on data from shippers in the Dallas, Houston, Galveston, and Lubbock markets. Total cost per bale to Southwestern shippers to all outlets was $19.13. The market trading area with the lowest average total cost per bale--$18.22--was the Houston-Galveston area. Of the 9 sales outlets included, total costs per bale ranged from a low of $7.52 for sales from the Houston-Galveston area to a high of $28.25 from the Lubbock area.

Cooper, Maurice R., Zolon M. Looney, and Shelby H. Holder. SHIPPERS' COSTS OF ASSEMBLING AND DISTRIBUTING MIDSOUTH COTTON, BY TYPES, MARKET TRADING AREAS, AND SALES OUTLETS, SEASON 1964-65. U.S. Dept. Agr., ERS-264 (1965), 2 pp., December 1965.

Estimates included are based on data from shippers in the Greenwood, Little Rock, Memphis, and New Orleans markets. Total cost per bale to shippers in all Midsouth areas combined was $13.92. The market trading area with the lowest average total cost per bale--$13.57--was the Memphis area. Of the 7 sales outlets reported, shippers' costs per bale ranged from about $10.71 for sales from Greenwood to $29.93 for sales from Little Rock-New Orleans.

Cooper, Maurice R., and Charles A. Wilmot. SHIPPERS' COSTS OF ASSEMBLING AND DISTRIBUTING WESTERN COTTON, BY TYPES, MARKET TRADING AREAS, AND SALES OUTLETS, SEASON 1964-65. U.S. Dept. Agr., Econ. Res. Serv., ERS-271 (1965), 2 pp., December 1965.

Estimates derived largely from shippers located in the Bakersfield, El Paso, Fresno, and Phoenix markets show that the weighted average total cost per bale for all Western areas combined was $21.31. The market trading area with the lowest average total cost per bale--$20.31--was the El Paso area. Of the sales outlets, total costs per bale ranged from $16.83 for sales from El Paso to $32.26 for sales from Phoenix.

Havas, Nick. PROFILE OF THE RETAIL FLORIST INDUSTRY 1964. U.S. Dept. Agr., Mktg. Res. Rpt. 741, 27 pp., December 1965.

The retail florist industry, consisting mainly of small businesses, accounts for sales exceeding $1 billion annually. Although only about one out of nine florists has annual sales of $100,000 or more, this small number accounts for nearly 40 percent of the industry's total sales. In addition to data on sales and number of establishments, the publication analyzes the industry's use of advertising and customer services, merchandising practices, loans, purchasing methods, and market outlets.

Moore, Charles V. ECONOMIES ASSOCIATED WITH SIZE, FRESNO COUNTY COTTON FARMS. Calif. Agr. Expt. Sta., Giannini Found. Res. Rpt. No. 285, 49 pp., November 1965. (Econ. Res. Serv. cooperating.)

This report presents estimated costs as related to farm size for two general soil types and associated adapted crops in Fresno County. Types of soils are (1) light sandy and (2) heavy clay and clay loam soils. Farmers can use the results of this report to make long-term plans in regard to the use of machinery, labor, and land.

[1] State publications may be obtained from the issuing agencies of the respective States.

U.S. GOVERNMENT PRINTING OFFICE : 1966 O - 214-162

Regier, D. W., R. N. Brown, R. W. Hexem, and W. P. Huth. MEAT IMPORT PROSPECTS OF THE EUROPEAN ECONOMIC COMMUNITY. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign 139, 28 pp., February 1966.

Since 1956, meat imports of the European Economic Community have been rising steadily; the sharpest increase occurred in 1964 when imports reached 1.4 million metric tons. Depending on factors such as the continuing vigorous growth of the internal economies of the EEC countries, meat imports are expected to be 1.5 million metric tons in 1966. This reflects a trade gap of 1.3 million metric tons, more than three times what it was in 1962.

Rice, Gabrielle P. CURRENT TRENDS IN INTERNATIONAL LIQUIDITY. U.S. Dept. Agr., Econ. Res. Serv., Foreign Gold and Exchange Reserves, 16 pp., December 1965.

In mid-1965, international reserves of gold and foreign exchange declined by one-half percent in developed countries, but increased 6 percent in less developed countries. For the most part, this increase reflects a more favorable trade position for the developing nations than in earlier periods. U.S. international reserves, primarily gold, continued to decline in the first half of 1965.

Sanderson, Agnes G. NOTES ON THE AGRICULTURAL ECONOMIES OF DEPENDENT TERRITORIES IN THE WESTERN HEMISPHERE AND PUERTO RICO. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign 145, 68 pp., December 1965.

Agriculture predominates in the 24 territories of the Western Hemisphere dependent on or associated with France, the Netherlands, the United Kingdom, or the United States. Agriculture employed about 36 percent of the labor force among a population of 5.3 million in 1963. The growth of manufacturing, mining, and tourist industries in the local economies, however, has reduced the relative importance of farming.

Twining, Carl R., and Peter L. Henderson. PROMOTIONAL ACTIVITIES OF AGRICULTURAL GROUPS. U.S. Dept. Agr., Mktg. Res. Rpt. 742, 32 pp., December 1965.

A survey of agricultural groups, including farmer cooperatives, commissions, councils, Federal or State boards, voluntary producer groups, and State agencies, was conducted during 1963-64. It showed that expenditures of voluntary producer-processors accounted for more than a third of total promotional expenditures of all groups. The largest single promotional expenditure was for advertising, mostly in magazines.

U.S. Department of Agriculture. CHANGES IN AGRICULTURE IN 26 DEVELOPING NATIONS, 1948 to 1963. Econ. Res. Serv., Foreign Devlpmt. and Trade Div., Foreign Agr. Econ. Rpt. 27, 134 pp., November 1965.

Between 1948 and 1963, crop output in 12 of the 26 developing nations increased more than 4 percent per year. These 12 were: Sudan, Mexico, Costa Rica, the Philippines, Tanganyika, Yugoslavia, Taiwan, Turkey, Venezuela, Thailand, Brazil, and Israel. The successes of these 12 countries indicate that underdeveloped countries generally can increase their per capita production of foods and fibers in the near future.

U.S. Department of Agriculture. FARM-MORTGAGE LENDING, EXPERIENCE OF 20 LIFE INSURANCE COMPANIES, FEDERAL LAND BANKS, AND FARMERS HOME ADMINISTRATION, January to June 1965 and Calendar Year 1965. Farm Prod. Econ. Div., Econ. Res. Serv., FML-15, 11 pp., January 1966.

During the first half of 1965, Federal land banks, 20 life insurance companies, and Farmers Home Administration closed farm-mortgage loans totaling \$1.3 billion. This amount was 24 percent more than in the first half of 1964. These 3 lenders held about 45 percent of total farm-mortgage debt on January 1, 1965.

U.S. DEPARTMENT OF AGRICULTURE
Economic Research Service
Washington, D.C. 20250

\- - -

Official Business

Postage and Fees Paid
U.S. Department of Agriculture

AGRICULTURAL ECONOMICS RESEARCH

Is published quarterly by the Economic Research Service, U.S. Department of Agriculture. Use of funds for printing this publication approved by the Director of the Bureau of the Budget (July 31, 1964).

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402. 25 cents a single copy, $1 a year domestic, $1.25 foreign.